Coordinates Determination of Underwater Sensors in Unparallel Situation by Using Cayley-Menger Determinant

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Abstract

Underwater Wireless Sensor Networks (UWSNs) are finding different applications for offshore exploration and ocean monitoring. In most of these applications, the network consists of significant number of sensor nodes deployed at different depths throughout the area of interest. The sensor nodes located at the sea bed cannot communicate directly with the nodes near the surface level; they require multi-hop communication assisted by appropriate routing scheme. In this paper, we present a method to determine the coordinates of underwater sensors from a single beacon node using Cayley-Menger determinant. But the problem is, Cayley-Menger determinant only we can use when the beacon plane and the sensors' plane are parallel with each other and there is no surety that the underwater sensors will be at the same depth always. Here we present a method that can find the underwater sensors coordinates when their depth positions are different and the plane of beacon and sensors are not parallel using Cayley-Menger determinant. In our presented method, we have presented that the Cayley-Menger determinant can be used in an unparallel situation. In this paper, first we have justified our result with the result of other papers. After that we have experimented with verifying different parameter which has also proved the capability of the presented method. Simulation results validate the proposed mathematical models by computing coordinates of sensor nodes with negligible errors.

Declaration

We hereby declare that, this project was done under CSE497 and has not been submitted elsewhere for requirement of any degree or diploma or for any purpose except for publication.

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Letter of Acceptance

We hereby declare that this thesis is from the student's own work and best effort of us, and all other sources of information used have been acknowledged. This thesis has been submitted with our approval.

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Abbreviation and Acronyms

WSN	Wireless Sensor Network
UWSN	Underwater Wireless Sensor Network
GPS	Global Positioning System
NDLP	Node Discovery and Localization Protocol
ALS	Area Localization Scheme
MLSL	Maximum-likelihood Source Localization Approach
LPS	Local Positioning System
RF	Radio Frequency
RSSI	Received Signal Strength Indicator
ISI	Inter Symbol Interference
TDOA	Time Difference of Arrival
TOA	Time of Arrival

Table of content

Chapter 1: Introduction

Chapter 2: Literature Review

Chapter 3: Proposed Method

List of Figures

List of figures **Page no.**

List of Tables

Chapter 1

Introduction

1.1 Underwater Wireless Sensor Network (UWSN)

The world is going smaller day by day and the invention of different technology is making our management and system easier, there is a huge contribution of wireless sensor network. In today’s world the effective implementation on the area of different sector of our system and management using WSN, it becomes the attraction of the researchers to make it more applicable in different way at low cost.

Wireless sensor network consists of some sensing devices that can communicate wirelessly and these devices can process and communicate with its peers. Precisely we can say that WSN is wireless network consisting of spatially distributed autonomous sensing device that can monitor physical and environmental conditions. These devices have the ability to coordinate and perform higher level of identification and task.

Its application has different dimension such as remote monitoring, heath care, mobile communication, surveillance, utilities and so on. It makes much easier as like maintaining our resource more intelligently. For example we can say the application in utilities-the electricity grid, street light and water municipals.In health care patient monitoring and such type of relevant work, for mobile communication base set up and monitoring its activity etc.

Underwater wireless sensor network is a very prominent and interesting research sector in recent years of WSN. Underwater Wireless Sensor Networks consists of a number of sensor nodes, stationary or mobile, connected wirelessly via acoustic communication modules deployed to monitor various events of interest collaboratively. The objective is achieved by having a set of autonomous devices in a network which can self-organize and adapt to deep-sea conditions. Underwater communication is mainly done using low frequency and low data rate acoustic modems with a set of nodes transmitting their data to a buoyant gateway that relays the data to nearest costal monitoring and control station.

Underwater wireless sensor networks consist of a variable number of sensors and vehicles that are deployed to perform collaborative monitoring tasks over a given area. Underwater Wireless Sensor Networks (UWSNs) are finding different applications for offshore exploration and ocean monitoring. In most of these applications, the network consists of significant number of sensor nodes deployed at different depths throughout the area of interest. The sensor nodes located at the sea bed cannot communicate directly with the nodes near the surface level; they require multi-hop communication assisted by appropriate routing scheme.

The majority of the earth's surface is covered with water. As more research is conducted on underwater systems, data collection and environment monitoring become major components. The characteristics of the underwater environment present researchers with many challenges, especially developing effective sensor communication and localization techniques.

Underwater wireless sensor networks have many potential applications. UWSN applications are rapidly gaining popularity for enabling advances in the area of ocean monitoring and observatory systems, deep sea surveillance, tracking of various entities of the aquatic environment, and unearthing resources. UWSNs find their application in fields like offshore oil and gas extraction, oil spills, military surveillance and reconnaissance, mine detection, pollution monitoring, natural calamities like tsunami and hurricane forecast, coral reef and habitat monitoring of marine life, and fish farming.

1.2 Problems of Underwater Wireless Sensor Network

The current pace of research in the area of underwater sensor networks (UWSNs) is slow due to the difficulties arising in transferring the state-of-the-art WSNs to their underwater equivalent. Maximum underwater deployments rely on acoustics for enabling communication combined with special sensors having the capacity to take on harsh environment of the oceans. However, sensing and subsequent transmission tend to vary as per different subsea environments; for example, deep sea exploration requires altogether a different approach for communication as compared to shallow water communication.

Underwater sensor networks are quite different from terrestrial sensor networks. In terrestrial wireless sensor networks, the nodes use Radio Frequency (RF) to establish the communication infrastructure. In underwater environments, due to water absorption, RF does not deliver the same performance. Compared to radio waves, sound has superior propagation characteristics in water, making it the preferred technology for underwater communications. However, since GPS may not work in underwater environments, acoustic signals bring many challenges to underwater sensor applications that require effective localization.

Terrestrial sensor networks employ electromagnetic waves but in underwater networks because of the characteristic (large delay, long distance of communication) of network, the communication is relied on physical means like acoustic sounds to transmit the signal. Traditional RF networks might not work efficiently in underwater networks.

Terrestrial networks are becoming inexpensive due to advancement in technology but underwater sensors are still expensive devices. This is due to the extra protection required for underwater environment and more complex transceivers needed.

Multipath propagation may be responsible for severe degradation of the UWSN’s communication signal, since it generates Inter Symbol Interference (ISI).The multipath geometry depends on the link configuration. Vertical channels are characterized by little time dispersion, whereas horizontal channels may have long multipath spreads. The extent of the spreading is a strong function of depth and the distance between the transmitter and the receiver.

The other obstacles of underwater sensor networks are the available bandwidth is severely limited, the underwater channel is impaired because of multi-path and fading, propagation delay in underwater is five orders of magnitude higher than in Radio Frequency (RF) terrestrial channels, and variable, high bit error rates and temporary losses of connectivity (shadow zones) can be experienced, underwater sensors are characterized by high cost because of extra protective sheaths needed for sensors and also relatively small number of suppliers are available. Underwater sensors are more prone to failures because of fouling and corrosion.

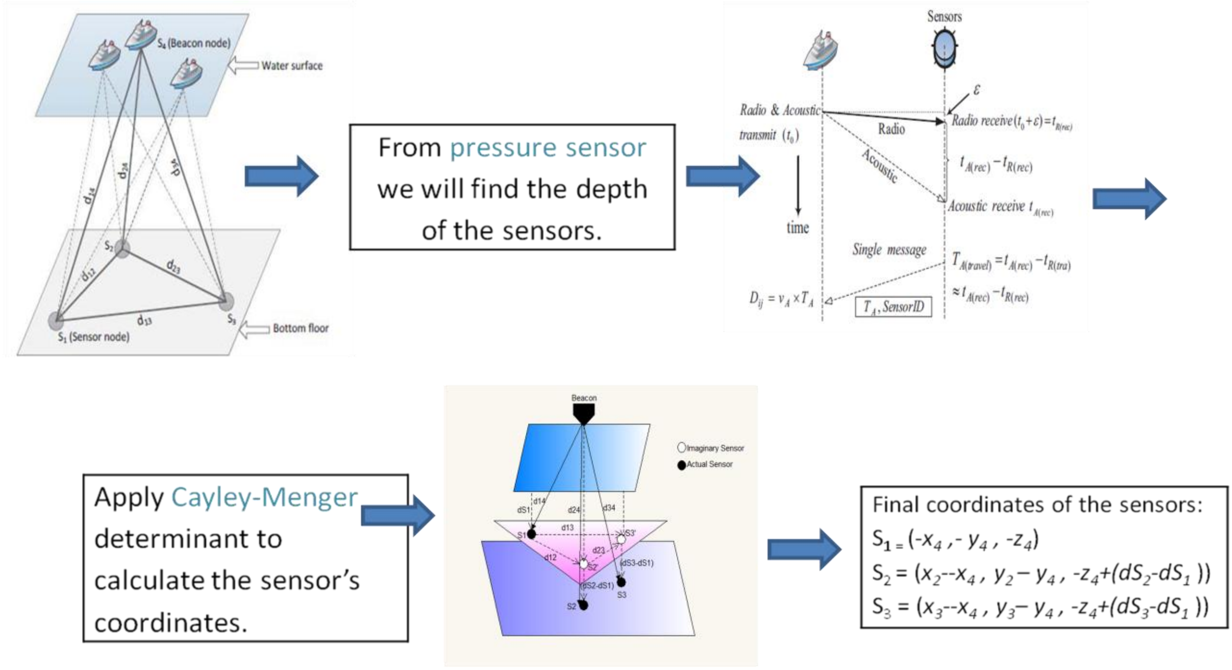
1.3 Objective

In a word, our objective is to apply the Cayley-Menger determinant to find the coordinates of the underwater sensors when the beacon plane and the sensors' made plane are not parallel with each other. Mentioned earlier that the Cayley-Menger determinant can only apply when the beacon plane and the sensors plane are parallel with each other.

Using our method (which is described in chapter 3) at first we will find the coordinates of the sensors from considering one of the sensors coordinate as the origin (0,0,0). Then finally we will compute the coordinates of the sensors from the beacon node using Cayley-Menger determinant according to the sensors' depth. As a result we will find the coordinates of the sensors using Cayley-Menger determinant. Following the result what we get from our simulation and will also compare the result with other paper.

1.4 Methodology

In this paper, we describe a closed-form solution to determine the coordinates of the underwater sensors having only one beacon node at the surface and the beacon node are assumed to be in a plane, which is unparallel to the plane created by the three sensors. Here we firstly calculate the depth of the sensors with respect to their pressure sensor's reading. Here we assumed that the distance measurement between the beacon and sensors are possible, we described distance measurement in chapter 3. For simplicity we assume an imaginary plane which is parallel to the beacon plane. Here we at first calculate the inter sensor distances using Cayley-Menger determinant and compute the coordinates of the sensors with known measurements. Then we linearly transform the origin of the sensor to the beacon. Then we calculate the coordinate of the sensors with respect to the beacon. Then at last we add the depth of the each sensor with their Z coordinates.



From pressure sensor we will find the depth of the sensors.

Apply Cayley-Menger determinant to calculate the sensor's coordinates.

Final coordinates of the sensors:

$S_1 = (-x_4, -y_4, -z_4)$

$S_2 = (x_2 - -x_4, y_2 - y_4, -z_4+(dS_2-dS_1))$

$S_3 = (x_3 - -x_4, y_3 - y_4, -z_4+(dS_3-dS_1))$

1.5 Results of Research

Here we mainly try to solve the problem of applying Cayley-Menger determinant in unparallel situation to determine the coordinate of the sensors.

At first we compute the coordinate of the sensors according to our proposed method using Cayley-Menger determinant. Then we have applied trilateration technique to determine the coordinate of the sensors with respect to the beacon. We implement our mathematical model and have found a good result. After that we use different parameter here. We achieved good result.

Chapter 2

Literature Review

2.1 Related works

Wireless information transmission through the ocean is one of the enabling technologies for the development of future ocean-observation systems and sensor networks. Applications of underwater sensing range from oil industry to aquaculture, and include instrument monitoring, pollution control, climate recording, prediction of natural disturbances, search and survey missions, and study of marine life.

Underwater wireless sensing systems are envisioned for stand-alone applications and control of autonomous underwater vehicles, and as an addition to cabled systems. For example, cabled ocean observatories are being built on submarine cables to deploy an extensive fibre-optic network of sensors (cameras, wave sensors and seismometers) covering miles of ocean floor.

Among the first underwater acoustic systems was the submarine communication system developed in the USA around the end of the Second World War. It used analogue modulation in the 8–11 kHz band . Research has since advanced, pushing digital modulation–detection techniques into the forefront of modern acoustic communications. At present, several types of acoustic modems are available commercially, typically offering up to a few kilobits per second (kbps) over distances up to a few kilometers. Considerably higher bit rates have been demonstrated, but these results are still in the domain of experimental research [3].

In underwater sensor networks (UWSNs), determining the location of every sensor is important and the process of estimating the location of each node in a sensor network is known as localization. While various localization algorithms have been proposed for terrestrial sensor networks, there are relatively few localization schemes for UWSNs. The characteristics of underwater sensor networks are fundamentally different from that of terrestrial networks. Underwater acoustic channels are characterized by harsh physical layer environments with stringent bandwidth limitations. The variable speed of sound and the long propagation delays under water pose a unique set of challenges for localization in UWSN.

WSNs generally utilize the unlicensed ISM bands for communications which are located at several frequency ranges varying from tens of MHz to several GHz values. It is not straight forward to use such high frequency signals in underwater because they are rapidly absorbed. In the underwater transmission the acoustic signal is mostly preferred instead of RF signal. The use of RF signals is considered to be limited to the nodes that are close to the surface only, although a recent study provides a different perspective on the use of RF signals under water. On the other hand, optical modems have been shown to achieve data rates reaching to M bit/s with ranges up to 100m, only in very clear water conditions. However, in practice it is hard to attain such conditions, and optical signals suffer from absorption and scattering at long ranges. For very short range underwater communications, i.e., 5m to 10m, optical modems have been utilized in[4].

2.2 Localization Techniques of UWSN

Underwater Wireless Sensor Networks (UWSNs) are expected to support a variety of civilian and military applications. Sensed data can only be interpreted meaningfully when referenced to the location of the sensor, making localization an important problem. While Global Positioning System (GPS) receivers are commonly used in terrestrial WSNs to achieve this, this is infeasible in UWSNs as GPS signals do not propagate through water. Acoustic communication is the most promising mode of communication underwater.

Localization methods for wireless sensor networks can be divided into two types:

- Range-based methods
- Range-free methods

2.2.1 Range-based methods

The range-based methods such as the received signal strength indicator (RSSI), time difference of arrival (TDOA) and time of arrival (TOA) use hardware to measure the distance information. These kinds of method have a higher accuracy, but they increase the network cost and energy consumption.

Received Signal Strength Indicator (RSSI)

Each ordinary node determines its distance from a reference node by measuring the Received Signal Strength and comparing it with a range dependent signal attenuation model. However, it is difficult to achieve accurate ranging when multipath and shadow fading effects exist. Since the path loss in underwater acoustic channels is usually time varying and multipath effect can result in significant energy fading, the RSSI method is not the primary choice for underwater localization.

Time Difference of Arrival (TDOA)

For indoor localization, the TDOA method utilizes the time difference between two different transmission mediums, namely, radio transmission and acoustic transmission, to calculate the distance between objects. Based on the two received signals, the distance to the transmitter can be determined. However, it is unsuitable for underwater localization because radio does not propagate well in water. Alternatively, the time difference of arrival between beacons from different reference nodes transmitted using acoustic signaling can be used in localization.

Time of Arrival (TOA)

The Time of Arrival (TOA) method performs ranging based on the relationship among transmission time, speed and distance. Most proposed range-based localization schemes use this method due to the limitations of the RSSI and TDOA-based approaches. However, TOA techniques may require time synchronization between network nodes.

As described earlier, the accuracy of range-based localization depends on the accuracy of range measurement, which could suffer from large errors due to node mobility as well as harsh underwater acoustic propagation environment. Hence, range-free schemes have been proposed that do not rely on range measurement for localization.

2.2.2 Range-free methods

The range-free methods use the connectivity of the network to locate the unknown nodes. The typical range-free methods mainly include the DV-HOP, Convex Programming and Centroid Localization algorithm. These methods have no additional hardware requirements, and they have lower energy consumption and shorter positioning time, but their accuracy is usually lower.

DV-HOP

DV-HOP assumes a heterogeneous network consisting of sensing nodes and anchors. Instead of single hop broadcasts, anchors flood their location throughout the network maintaining a running hop-count at each node along the way. Nodes calculate their position based on the received anchor locations, the hop-count from the corresponding anchor, and the average distance per hop; a value obtained through anchor communication.

Chapter 3

Proposed Method

3.1 Problem statement

In this paper, to determine the coordinates of the submerged sensors, our proposed method assumes at least three sensors and a floating beacon. It is also assumed that the distance measurement between the beacon and sensors are possible (which is described later part in this chapter). In the marine environment, a boat or a buoy can be used as a beacon and sensors could be deployed in the water. While measuring the multiple distances between the beacon and sensors, those locations of the beacon are assumed to be in a plane, which is approximately parallel to the plane created by the three sensors (as shown in Figure. 3.1). But in the reality the beacon and the sensor created plane are not parallel with each other. Because the depth location of the sensors are different. So to make the method simple we assume an imaginary sensors created plane which is parallel to the beacon plane [1].

As the general properties of a transducer, beacon has the capability of generating radio and acoustic signal, whereas sensors might have the restricted capability of receiving the radio and acoustic signal for timing purpose as well as it is enabled with acoustic transmission. The sensors also have installed pressure sensors by which they can measure the pressure of the water. A solvable configuration of one beacon with three submerged sensors is denoted in Figure. 3.2. Our proposed mechanism exploits the advantage of both radio and acoustic signal propagation in sea water in 1.8-323m depth. Since most of the marine explorations take place in shallow water, our proposed model has wide ranging practical applications.

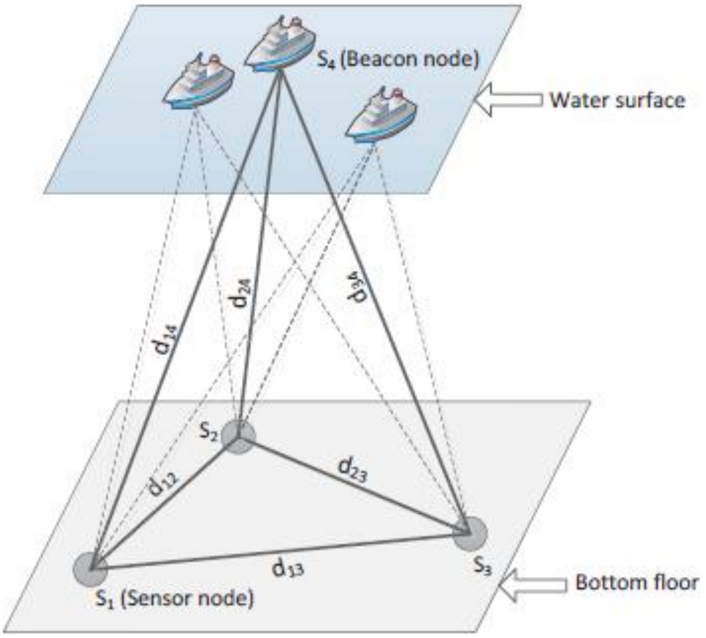


Figure. 3.1 A solvable configuration of one beacon with three submerged sensors when two planes are parallel with each other

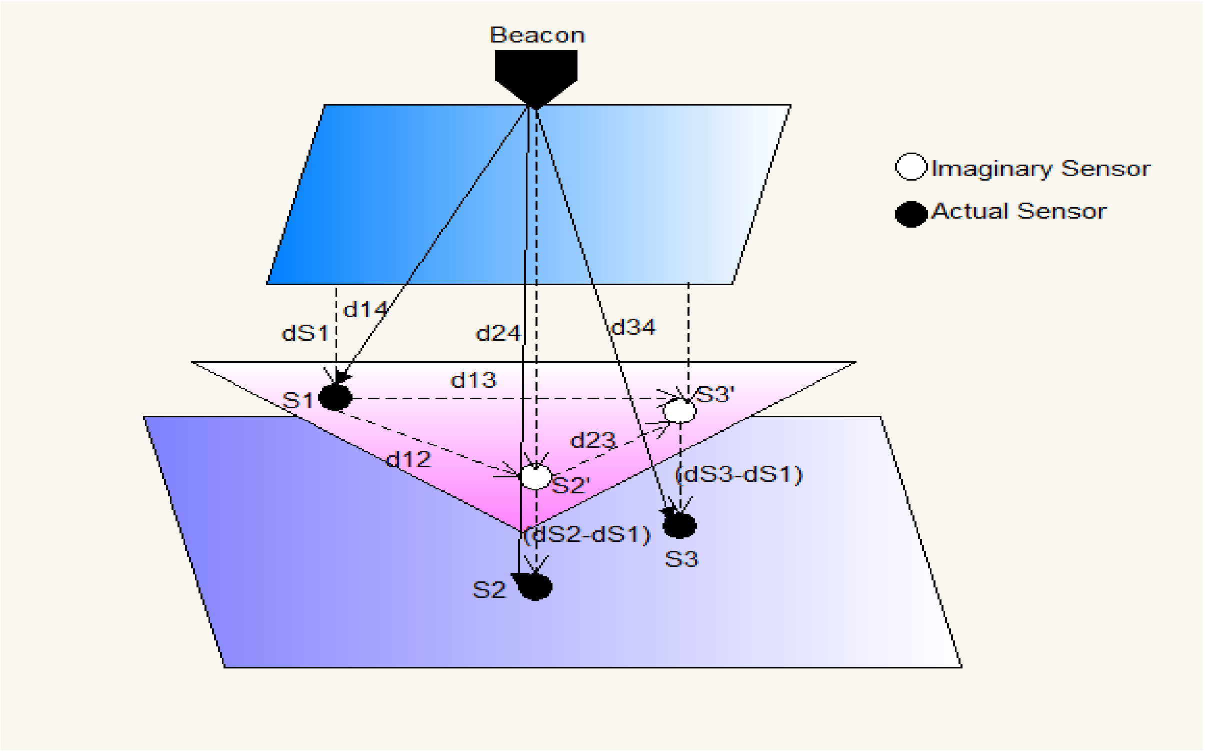


Figure 3.2 The solvable configuration of one beacon with three submerged sensors when two planes are unparallel with each other

3.2 Environmental Limitations

Normally, underwater environment is more adverse than terrestrial environment; despite those limitations, it poses some merits that could be exploited in determining coordinates. Water body is relatively more homogeneous because the usual obstacles present in water are smaller in size than that of in terrestrial environment. The region of interest on the ground is more likely occupied with buildings and trees which are the major factors for multipath propagation [1].

Regarding signal propagation in water, acoustic signal propagates much further compare to radio signal; however, Figure. 3.1. A solvable configuration of one beacon with three submerged sensors when two planes are parallel with each other the speed of the acoustic signal is much slower than that of radio signal. Table 3.1 shows some limitations and typical measurements for radio and acoustic signals.

Table 3.1 Properties of Radio and Acoustic signal

	Radio signal		Acoustic signal	
	Vacuum	Water	Vacuum	Water
Velocity	3×108 m/s	≈2.25×106 m/s	≈1500 m/s	VA
Range	-	1.8-323 m	-	1 - 100 km

The main environmental variable that we assume in our method to determine distances is the speed of acoustic signals in water. It depends on the temperature, salinity and permeability. How the speed of acoustic will vary because of aforesaid factors is not considered in this study, but our mathematical model assumes it as a variable *vA*.

3.3 Depth Measurement

Here we assume that the sensors have pressure sensors. So that we can calculate the depth of the sensors under the water from their pressure's readings . The equation that convert the pressure into depth is given below:

depth (meters) = $[(((-1.82\text{x}10^{-15} * p + 2.279\text{x}10^{-10}) * p - 2.2512\text{x}10^{-5}) * p + 9.72659) * p] / g$

Where,

p = pressure (decibars)

g = gravity (m/sec^2)

$x = [\sin(\text{latitude} / 57.29578)]^2$

From this equation, we calculate the depth of the sensors and find the minimum depth of them. Here we have considered the minimum depth sensor as the origin (0,0,0) and the first sensor S_1.

3.4 Distance Measurement

Despite the limitation of both radio and acoustic signal propagation in water in different aspect, we will exploit each of their merits in our proposed method to increase the accuracy of the distance measurements. Differential speed between radio and acoustic signals will be used to calculate the distance, while acoustic signal will be used for communication purposes. This method will require a short communication in between the beacon and sensor nodes.

Even though the speed of radio signal is slightly less than that of in the vacuum, considering the problem domain, the speed variation will not have significant impact on the proposed localization method. Moreover, the speed of acoustic signal, which varies due to different environmental factors, is the main variable that we need to use for coordinate determination [1].

We mentioned earlier that the beacon can produce radio and acoustic signals simultaneously. The distance measurement calculation mentioned below :

(i) Simultaneous generation of radio and acoustic signals by the beacon S_j, j = 4,5.... at t_0.

(ii) For any submerged sensors S_i , i = 1,2,3

   a. Sensors receive the radio signal at $t_{R(rec)} = t_0 + \varepsilon$

   b. Sensor receives the acoustic signal at $t_{A(rec)}$; here $t_{A(rec)} \gg t_{R(rec)}$

(iii) Time taken for the acoustic signal to travel from beacon to sensor is :

$$T_{ij(Travel),i=1,2,3;j=4,5,6.....} = t_{A(rec)} - t_{A(tra)}$$

$$= t_{A(rec)} - t_{A(tra)} \text{ but } t_{A(tra)} = t_{R(tra)}$$

$$\therefore T_{ij(Travel)} \approx t_{A(rec)} - t_{R(rec)}$$

$$\text{cause } t_{R(rec)} = t_0 + \varepsilon \approx t_{R(tra)}$$

(iv) Sensor nodes send back the time $T_{ij(travel)}$ with individual sensor's ID to the beacon using acoustic signal.

(v) Beacon nodes compute the distance between the beacon and sensors: $d_{ij=}v_A * T_{ij(Travel)}$ here, v_A is average acoustic signal speed.

Being aware of the limitations of radio and acoustic signals in water, each of its merit has been used in our proposed method to determine the distances in the problem domain. The method is relatively simple but precise enough when both the beacon and sensors are capable of transmitting/receiving radio and acoustic signals. To be precise, the beacon should be capable of acoustic (transmit (*Tx*) & receive (*Rx*)) and radio (transmit (*Tx*)) only. On the other hand, sensors should be capable of acoustic (transmit (*Tx*) & receive (*Rx*)) and radio (receive (*Rx*)) only. Considering most of the practical applications, this assumption is considered pragmatic [1].

For the acoustic velocity, we are using *vA* in our mathematical model, which depends on several factors and how these factors affect *vA* is left for future investigation. However, in our approach, we are able to measure the time in between transmission and reception using radio signals more accurately. Furthermore, our model has very low overhead. Figure. 3.3 shows the sequence of action that each node performs; where at the end only one message with the value of *Tij* (*travel*) and sensor ID is transferred via acoustic signal from sensor to beacon for distance calculation.

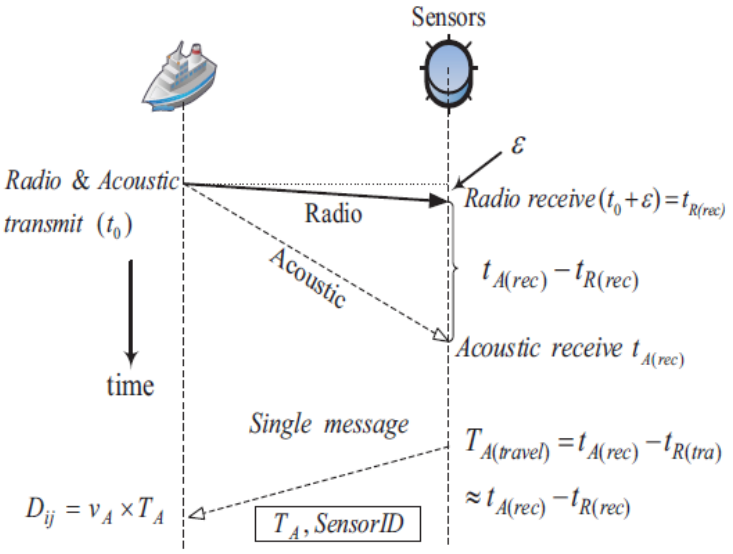


Figure. 3.3 Message transmission for distance calculation

3.5 Coordinates of the Sensors

The objective of localization algorithms is to obtain the exact position or coordinates of all the sensor nodes by measuring distances between beacon and them. Only measurements available here to compute is the distance and typically it is considered as optimization problem where objective functions to be minimized have residuals of the distance equations. The variables of any localization problem are the 3D coordinates of the nodes. In principle more number of distance equations are needed than number of variables to solve this kind of problem. However, this approach known as degree of freedom analysis may not guarantee the unique solution in a nonlinear system [1].

Trilateration or multilateration techniques that are nonlinear system usually used to determine the location or coordinates of the sensors in partial or full. According to Guevara et al. [10] the convergence of optimization algorithms and Bayesian methods depend heavily on initial conditions used and they circumvent the convergence problem by linearizing the trilateration equations.

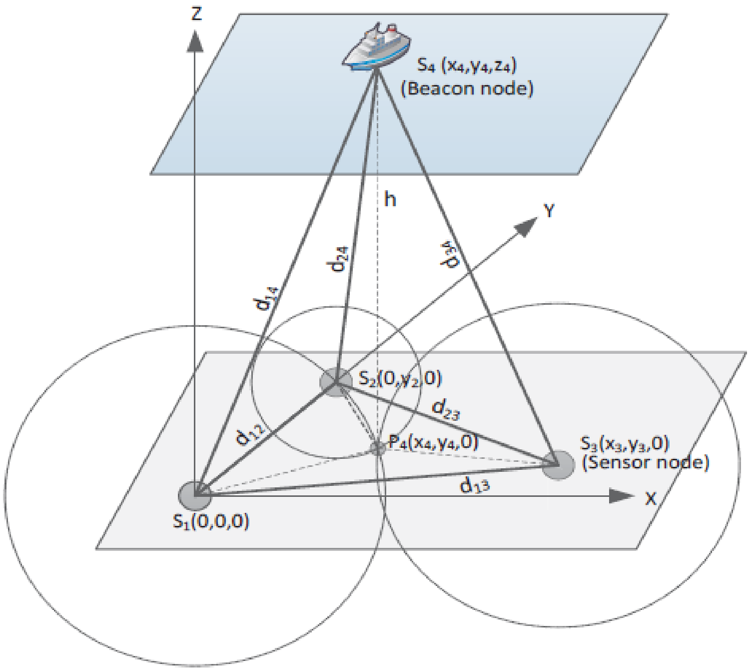


Figure. 3.4 Coordinates determinations

Figure. 3.4 shows the initial subset composed of the beacon node S_j , j = 4,5....9 and three sensor nodes S_i , i =1,2,3. Without loss of generality, a coordinate system can be defined using one of the sensor nodes Si , i =1,2,3. as the origin (0,0,0) of the coordinate system. Now the trilateration equations can be written as a function of two groups of distance measurements. The distance between beacon and sensors d_{14}, d_{24}, d_{34}.... which are measured data, and inter sensor distances d_{12}, d_{13}, d_{23} and the volume of tetrahedron V_t (formed by the beacon and sensors), are unknown. By expanding and grouping known-unknown variables of (1), we obtain ;

$$d_{34}^2(d_{12}^2 - d_{23}^2 - d_{13}^2) + d_{14}^2\left(\frac{d_{23}^4}{d_{12}^2} - d_{23}^2 - \frac{d_{13}^2 d_{23}^2}{d_{12}^2}\right) + d_{24}^2\left(\frac{d_{13}^4}{d_{12}^2} - \frac{d_{13}^2 d_{23}^2}{d_{12}^2} - d_{13}^2\right) - (d_{14}^2 d_{24}^2 + d_{14}^2 d_{34}^2 - d_{24}^2 d_{34}^2 - d_{14}^4)\frac{d_{23}^2}{d_{12}^2} - (d_{34}^2 d_{24}^2 - d_{14}^2 d_{34}^2 + d_{14}^2 d_{24}^2 - d_{24}^4)\frac{d_{13}^2}{d_{12}^2} + \left(144\frac{V_t^2}{d_{12}^2} + d_{13}^2 d_{23}^2\right) = (d_{24}^2 d_{34}^2 - d_{34}^2 + d_{14}^2 d_{34}^2 - d_{14}^2 d_{24}^2)$$

Here,

$(d_{12}^2 - d_{23}^2 + d_{13}^2)$, $\left(\frac{d_{23}^2}{d_{12}^2} - d_{23}^2 - \frac{d_{13}^2 d_{23}^2}{d_{12}^2}\right)$, $\left(\frac{d_{13}^4}{d_{12}^2} - \frac{d_{13}^2 d_{23}^2}{d_{12}^2} - d_{13}^2\right)$, $\frac{d_{23}^2}{d_{12}^2}$, $\frac{d_{13}^2}{d_{12}^2}$ and $\left(144\frac{V_t^2}{d_{12}^2} + d_{13}^2 d_{23}^2\right)$ are known terms.

The above expression can be written as follows,

$$d_{14}^2 X_1 + d_{24}^2 X_2 + d_{34}^2 X_3 - (d_{14}^2 - d_{34}^2)(d_{24}^2 - d_{14}^2)X_4 - (d_{24}^2 - d_{14}^2)(d_{34}^2 - d_{24}^2)X_5 + X_6 = (d_{24}^2 - d_{34}^2)(d_{34}^2 - d_{14}^2) \qquad \text{…….} (1)$$

Based on the local positioning system configuration of Figure. 3.4, we need to write equations that will include all known and unknown distances. For that matter, we express the volume of tetrahedron V_t using Cayley-Menger determinant as following:

$$288\, V_t^2 = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & d_{12}^2 & d_{13}^2 & d_{14}^2 \\ 1 & d_{12}^2 & 0 & d_{23}^2 & d_{24}^2 \\ 1 & d_{13}^2 & d_{23}^2 & 0 & d_{34}^2 \\ 1 & d_{14}^2 & d_{24}^2 & d_{34}^2 & 0 \end{vmatrix} \qquad \text{…..} (2)$$

The Equation (2) in fact resembles the linear form of $a_1 x_1 + a_2 x_2 + \ldots.. + a_n x_n = b1$. As we have six unknown in (1), we need at least six measurements, which could be done following the same procedure described earlier steering the beacon node $S\ j$, j=4,5....9 to six different locations and measuring the distances in the vicinity of $S4$. Finally, we get m-linear equations of the form;

$$a_{11}x_1+a_{12}x_2+\ldots\ldots+a_{1n}x_n=b_1,$$

$$a_{21}x_1+a_{22}x_2+\ldots\ldots+a_{2n}x_n=b_2,$$

$$\vdots$$

$$a_{m1}x_1+a_{m2}x_2+a_{mn}x_n=b_m \qquad \ldots\ldots(3)$$

If we omit reference to the variables, then system of equations in (3) can be represented by the array of all coefficients known as the augmented matrix of the system, where the first row of the array represents the first linear equation and so on. That could then be expressed in $AX=b$ linear form. Then, the system of equations can be written as:

$$A=\begin{bmatrix} d_{14}^2 & d_{24}^2 & d_{34}^2 & -(d_{14}^2-d_{34}^2)(d_{24}^2-d_{14}^2) & -(d_{24}^2-d_{14}^2)(d_{34}^2-d_{24}^2) & 1 \\ d_{15}^2 & d_{25}^2 & d_{35}^2 & -(d_{15}^2-d_{35}^2)(d_{25}^2-d_{15}^2) & -(d_{25}^2-d_{15}^2)(d_{35}^2-d_{25}^2) & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ d_{19}^2 & d_{29}^2 & d_{39}^2 & -(d_{19}^2-d_{39}^2)(d_{29}^2-d_{19}^2) & -(d_{29}^2-d_{19}^2)(d_{39}^2-d_{29}^2) & 1 \end{bmatrix}$$

$$X=\begin{bmatrix} (\frac{d_{23}^4}{d_{12}^2}-d_{23}^2-\frac{d_{13}^2 d_{23}^2}{d_{12}^2}) \\ (\frac{d_{13}^4}{d_{12}^2}-\frac{d_{13}^2 d_{23}^2}{d_{12}^2}-d_{13}^2) \\ (d_{12}^2-d_{23}^2-d_{13}^2) \\ \frac{d_{23}^2}{d_{12}^2} \\ \frac{d_{13}^2}{d_{12}^2} \\ (144\frac{V_t^2}{d_{12}^2}+d_{13}^2 d_{23}^2) \end{bmatrix} \qquad b=\begin{bmatrix} (d_{24}^2-d_{34}^2)(d_{34}^2-d_{14}^2) \\ (d_{25}^2-d_{35}^2)(d_{35}^2-d_{15}^2) \\ \vdots \\ (d_{29}^2-d_{39}^2)(d_{29}^2-d_{19}^2) \end{bmatrix}$$

From the above representation, after finding X_1, X_2 , X_3, X_4 , X_5 and X_6 we calculate d_{12} , d_{13} and d_{23} as follows:

$$d_{12}^2=\frac{X_3}{(1-X_4-X_5)},\ d_{13}^2=\frac{X_3\,X_5}{(1-X_4-X_5)},\ d_{23}^2=\frac{X_3\,X_4}{(1-X_4-X_5)}$$

If we let the coordinates of the submerged sensors S_1, S_2 and S_3 are (0,0,0), (0, y_2 ,0) and (x_3, y_3 ,0) respectively, then the inter-sensor distances could be written with respect to coordinates of the sensors as follows:

$$d_{12}^2 = y_2^2 \, , \; d_{13}^2 = x_3^2 + y_3^2 \, , \; d_{23}^2 = x_3^2 + (y_3 - y_2)^2$$

From the above values the unknown variables can be computed as follows:

$$y_2 = d_{12}, \; y_3 = \frac{d_{12}^2 + d_{13}^2 - d_{23}^2}{2d_{12}}, \; x_3 = \sqrt{\left(d_{13}^2 - \left(\frac{d_{12}^2 + d_{13}^2 - d_{23}^2}{2d_{12}}\right)^2\right)}$$

where d_{12}, d_{13} and d_{23} are known computed distances Table 3.2 summarizes the coordinates of the sensors for this system.

Table 3.2 Coordinates of the sensors with known measurements

Sensors	**Coordinates**
S_1	(0,0,0)
S_2	$(0, d_{12}, 0)$
S_3	$\left(\sqrt{\left(d_{13}^2 - \left(\frac{d_{12}^2 + d_{13}^2 - d_{23}^2}{2d_{12}}\right)^2\right)}, \frac{d_{12}^2 + d_{13}^2 - d_{23}^2}{2d_{12}}, 0\right)$

3.6 Coordinates of the Sensors with respect to the Beacon

Up to now we have been able to determine the coordinates of the sensor nodes with respect to S_1 . In order to find the coordinate with respect to the beacon node we follow the following steps.

We assume that with the use of the pressure sensor of the sensors , the depth h can be measured for each sensor. We also assume that there is a imaginary plane which is parallel to the beacon plane and its all coordinated of Z are 0. Here we have considered the imaginary plane to calculate the coordinate of the sensors.

After measuring the vertical distance h in between the beacon node S_4 (x_4 , y_4 , z_4) and the XY plane, we can assume the projected coordinate of the beacon node S_4 (x_4 , y_4 , z_4) on the plane XY is P_4 (x_4, y_4,0) . To find x_4 and y_4 , we can apply trilateration in the following manner assuming the distances between S_1, S_2, S_3 and P_4 are D_{14} , D_{24} and D_{34} respectively.

$$\mathrm{D}_{14}^2 = \mathrm{x}_4^2 + \mathrm{y}_4^2 \quad \text{..........(4)}$$

$$\mathrm{D}_{24}^2 = \mathrm{x}_4^2 + (\mathrm{y}_4 - \mathrm{y}_2)^2 \quad \text{..........(5)}$$

$$D_{34}^2 = (\mathrm{x}_4 - \mathrm{x}_3)^2 + (\mathrm{y}_4 - \mathrm{y}_3)^2 \quad \text{........(6)}$$

From equation (4), (5) and (6) we obtain the projected beacon's coordinates P_4 (x_4 , y_4 , 0).

$$\mathrm{x}_4 = \frac{1}{2d_{12}} \sqrt{(4d_{12}^2 D_{14}^2 - (D_{14}^2 - D_{24}^2 + d_{12}^2)^2)}$$

$$\mathrm{y}_4 = \frac{1}{2d_{12}} \ (\mathrm{D}_{14}^2 - \mathrm{D}_{24}^2 + d_{12}^2)$$

As d_{14} , d_{24} and d_{34} are hypotenuse of the $\Delta S_1P_4S_4$, $\Delta S_2P_4S_4$ and $\Delta S_3P_4S_4$ respectively , so it is possible to obtain D_{14} , D_{24} and D_{34} using Pythagorean Theorem. So the coordinate of the beacon node S_4 (x_4 , y_4 , z_4) would be S_4 (x_4 , y_4 , h) where all the elements are known.

$$\therefore S_4\ (x_4\ ,\ y_4\ ,\ 0) = \left[\left(\sqrt{\tfrac{1}{2d}\ (2d_{12}D_{14}^2 - D_{14}^2 + D_{24}^2 + d_{12}^2)}\right), \left(\tfrac{1}{2d_{12}}\ (\mathrm{D}_{14}^2 - \mathrm{D}_{24}^2 + d_{12}^2)\right), h\right]$$

If the origin of the Cartesian system is transferred on to the coordinate of the beacon node, then it is possible to find the coordinates of other sensors with respect to the beacon node S_4. A linear transformation would give the results as in Table 3.3.

Table 3.3 Coordinates of the sensors with respect to beacon for parallel situation

Sensors	Coordinates	Sensors	Coordinates
S_4	(0, 0, 0)	S_2	$(x_2 - -x_4\ ,\ y_2 - y_4\ ,\ -z_4)$
S_1	$(-x_4\ ,-\ y_4\ ,\ -z_4)$	S_3	$(x_3 - -x_4\ ,\ y_3 - y_4\ ,\ -z_4)$

As our purpose is to calculate the sensors' coordinate for unparallel situation so now we will calculate the sensors' coordinate according to their depth. The depth of the sensor is different in the underwater situation. As we know that the depth of the sensor is presented by the Z-axis so simply we just add the depth value with the z-coordinate of the sensor which we have shown at the Table 3.4. The final coordinate of the sensors are given in the Table 3.4 for unparallel situation according to the figure.3.2 (a).

Table 3.4 Coordinates of the sensors with respect to beacon for unparallel situation

Sensors	Coordinates	Sensors	Coordinates
S_4	(0, 0, 0)	S_2	$(x_2 - -x_4\ ,\ y_2 - y_4\ ,\ -z_4 + (d_B - d_A))$
S_1	$(-x_4\ ,-\ y_4\ ,\ -z_4 + d_A)$	S_3	$(x_3 - -x_4\ ,\ y_3 - y_4\ ,\ -z_4 + (d_C - d_A))$

Chapter 4

Computational Results & Analysis

A simulation of the proposed method to determine the coordinates of submerged sensors as described in chapter 3 was performed to verify the method. The experiment was designed based on 3-D space. As the distance measurement between beacon and sensors is possible. So at first we assume 3 depth position of the sensors. Then we find the minimum depth of the sensors. Then with respect to the minimum depth of the sensors three sensors are placed at (0, 0, 0), (0, 60, 0) and (85, 90, 0) and a floating bacon randomly moved in a plane which is parallel to the imaginary XY plane where the sensors are in 3D- space. The coordinates of the sensors are randomly chosen. Z-coordinates of sensors is always kept zero to satisfy that all sensors are situated in same plane and for computational simplicity one of the coordinates are placed at the origin. After that we are calculated the final coordinate of the sensors from the beacon according to our proposed method. This proposed method has been simulated using Matlab(v- 2014b). To simulate the proposed method 50 datasets were taken. Each datasets contains six different positions of the beacon to get distance measurement and it has been randomly moved around to six different coordinates. Gaussian noise has been added While calculating the true Euclidian distance from six different bacon nodes to sensors S_1, S_2 and S_3. The simulation result of our proposed method is given below:

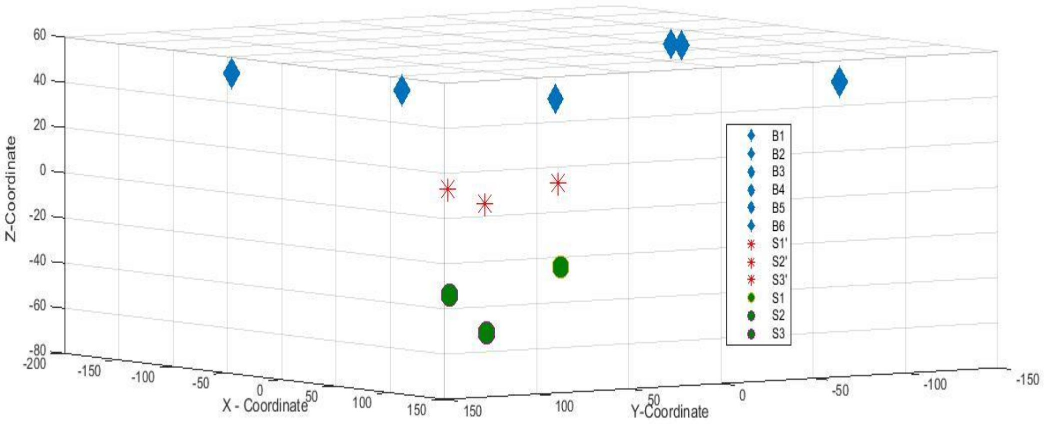


Figure. 3.5 Calculated sensors positions with proposed method

To calculate the coordinates of sensors we need the inner distances between sensors S_1, S_2 and S_3. After solve the linear equation (3) which is formed by Cayley-Menger determinant equation (2) we find the inner distances between sensors S_1, S_2 and S_3. These distances may contain some Gaussian error because of it calculated from the Gaussian error distances between bacon and sensors.

Errors in coordinates for sensors S_2 and S_3 are shown in Fig. 3.6 and Fig. 3.7 respectively for 50datasets. It should be noted that sensor S_1 is placed at the reference coordinate (0,0,0); hence producing no error in coordinate determination for S_1 , moreover S_2 and S_3 are computed with respect to S_1 .

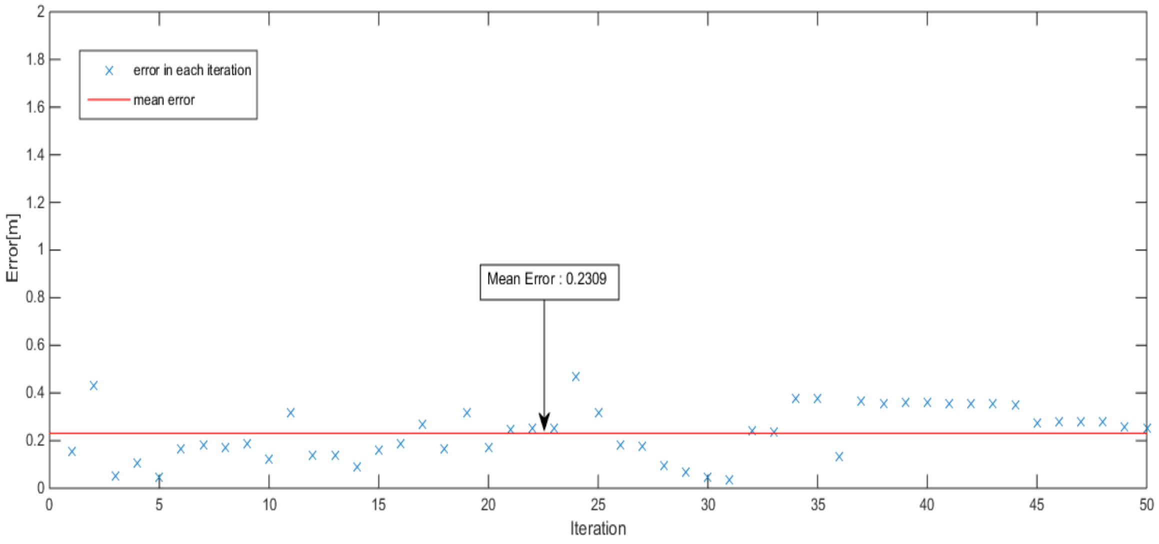


Figure. 3.6 Mean error for S_2

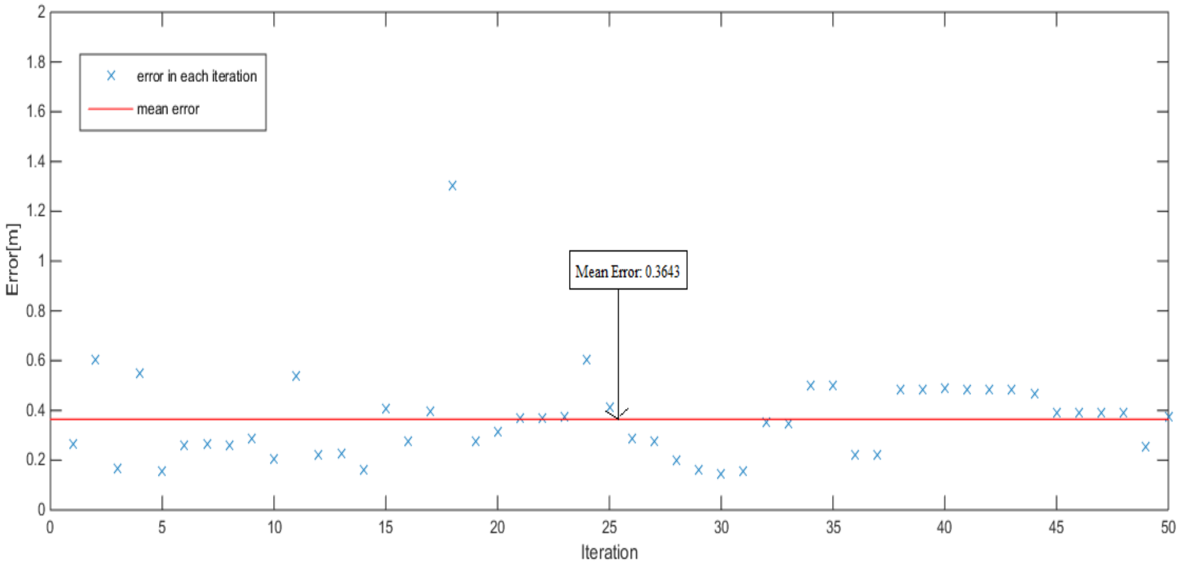


Figure. 3.7 Mean error for S_3

Table 3.5 compares the positional error of sensor *S*2 and *S*3 when distances between the beacon and sensors are true Euclidean as well as with Gaussian noise.

Table 3.5 Generation of Errors

Sensors (Unknown coordinates)	Mean error(m) (with Gaussian Noise)
S_2	0.2309
S_3	0.3643

Chapter 5

Conclusion and Future Work

We have described a measurement technique in under water sensor localization using Cayley-Menger determinant when the beacon plane and sensors created plane are unparallel. We use multilateration technique to determine the location of the sensors with respect to beacon node where distance between them is measured considering the acoustic and radio signal. Cayley–Menger determinant is used to determine the nodes coordinates, it reduces the impact of distance measurement error on the location estimation. Simulation result validate the mathematical model by computing coordinates of sensors with negligible error. Therefore the coordinates are acceptable error range after adding Gaussian noise into the distance measurement.

Our future plan to do about the mobility of the sensors because when we take the reading of the sensor it does not stay at the same place always.

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Appendix

Data Set

Sl	Bacon Nodes (from six different position)	Sensor Nodes				Sensor Nodes with Gaussian Error				Distance Error From Original Position
			X	Y	Z		X	Y	Z	
1	B1 = [150 90 50] B2 = [-100 -120 50] B3 = [-80 130 50] B4 = [140 -70 50] B5 = [60 120 50] B6 = [-90 -130 50]	S1	0	0	0	S1	0	0	0	0
		S2	0	60	0	S2	0	60.1550	0	0.1550
		S3	85	90	0	S3	85.1453	90.2218	0	0.2651
2	B1 = [50 50 50] B2 = [-70 -120 50] B3 = [-80 110 50] B4 = [90 -130 50] B5 = [60 100 50] B6 = [-100 -120 50]					S1	0	0	0	0
						S2	0	60.4310	0	0.4310
						S3	85.1480	90.5835	0	0.6020
3	B1 = [100 90 50]; B2 = [-100 -120 50]; B3 = [-80 130 50]; B4 = [140 -70 50]; B5 = [60 110 50]; B6 = [-90 -130 50];					S1	0	0	0	0
						S2	0	60.0512	0	0.05129
						S3	85.0285	90.1651	0	0.1676
4	B1 = [100 90 50]; % B2 = [-110 -120 50]; B3 = [-80 130 50];					S1	0	0	0	0
						S2	0	60.1070	0	0.1070

	B4 = [140 -70 50]; B5 = [60 110 50]; B6 = [-90 -130 50];		S3	84.5421	90.3025	0	0.5488
5	B1 = [100 90 50]; % B2 = [-110 -120 50]; B3 = [-80 130 50]; B4 = [140 -80 50]; B5 = [60 110 50]; B6 = [-100 -130 50];		S1	0	0	0	0
			S2	0	60.0462	0	0.04621
			S3	84.9840	90.1556	0	0.1564
6	B1 = [100 90 50]; % B2 = [-110 -120 50]; B3 = [-80 130 50]; B4 = [140 -80 50]; B5 = [160 -110 50]; B6 = [-100 -130 50];		S1	0	0	0	0
			S2	0	60.1679	0	0.1679
			S3	85.1109	90.2359	0	0.2607
7	B1 = [100 90 50]; % B2 = [-110 -120 50]; B3 = [-80 130 50]; B4 = [40 -80 50]; B5 = [160 -110 50]; B6 = [-90 -130 50];		S1	0	0	0	0
			S2	0	60.1801	0	0.1801
			S3	85.1102868	90.2424	0	0.2663
8	B1 = [100 90 50]; % B2 = [-110 -120 50]; B3 = [-180 130 50]; B4 = [140 -80 50]; B5 = [160 -110 50]; B6 = [-90 -130 50];		S1	0	0	0	0
			S2	0	60.1721	0	0.1721
			S3	85.0841	90.2468		0.26075
9	B1 = [100 90 50]; % B2 = [-119 -120 50]; B3 = [-180 130 50]; B4 = [140 -145 50]; B5 = [160 -110 50]; B6 = [-90 -130 50];		S1	0	0	0	0
			S2	0	60.1871	0	0.1871
			S3	85.2042	90.2048	0	0.2893

10	B1 = [100 90 50]; % B2 = [-119 -120 50]; B3 = [-180 156 50]; B4 = [140 -145 50]; B5 = [160 -115 50]; B6 = [-90 -170 50];		S1	0	0	0	0
			S2	0	60.1195	0	0.1195
			S3	85.1026	90.1779	0	0.20544
11	B1 = [102 90 50]; % B2 = [-119 -120 50]; B3 = [-180 156 50]; B4 = [140 -148 50]; B5 = [167 -115 50]; B6 = [90 -170 50];		S1	0	0	0	0
			S2	0	60.3162		0.3162
			S3	85.5024	90.1943		0.5387
12	B1 = [102 111 50]; % B2 = [-119 -122 50]; B3 = [-180 136 50]; B4 = [-140 -148 50]; B5 = [167 -115 50]; B6 = [90 -170 50];		S1	0	0	0	0
			S2	0	60.1386	0	0.1386
			S3	85.1217	90.1858		0.2221
13	B1 = [102 111 50]; % B2 = [-111 -122 50]; B3 = [-188 133 50]; B4 = [-144 -144 50]; B5 = [166 -115 50]; B6 = [99 -177 50];		S1	0	0	0	0
			S2	0	60.1403		0.14035
			S3	85.1257	90.1865		0.2249
14	B1 = [102 161 50]; % B2 = [-111 -122 50]; B3 = [208 -119 50]; B4 = [-104 -167 50]; B5 = [105 -195 50]; B6 = [199 -197 50];		S1	0	0	0	0
			S2	0	60.0882		0.08823
			S3	85.0798	90.1372		0.1587

15	B1 = [-102 161 50]; % B2 = [111 -122 50]; B3 = [89 -119 50]; B4 = [194 -137 50]; B5 = [105 -165 50]; B6 = [199 -117 50];		S1	0	0	0	0
			S2	0	60.1578	0	0.1578
			S3	85.2809	90.2933	0	0.40617
16	B1 = [-55 71 50]; % B2 = [101 -102 50]; B3 = [89 -100 50]; B4 = [94 -130 50]; B5 = [-105 -165 50]; B6 = [98 115 50];		S1	0	0	0	0
			S2	0	60.1866	0	0.1866
			S3	85.1060	90.2527	0	0.2740
17	B1 = [-55 50 50]; % B2 = [-101 -102 50]; B3 = [87 -100 50]; B4 = [90 -130 50]; B5 = [-105 -165 50]; B6 = [-98 -115 50];		S1	0	0	0	0
			S2	0	60.2700	0	0.2700
			S3	85.2203	90.3314	0	0.3980
18	B1 = [55 60 50]; % B2 = [101 102 50]; B3 = [87 -100 50]; B4 = [90 -130 50]; B5 = [85 165 50]; B6 = [98 -195 50];		S1	0	0	0	0
			S2	0	59.8356	0	0.1643
			S3	85.0275	91.3029	0	1.3032
19	B1 = [55 -60 50]; % B2 = [-101 102 50]; B3 = [-87 -100 50]; B4 = [-92 -130 50]; B5 = [-93 165 50]; B6 = [-102 195 50];		S1	0	0	0	0
			S2	0	59.6839	0	0.3160
			S3	84.7676	89.8483	0	0.27750
20	B1 = [55 -56 50]; B2 = [-101 102 50]; B3 = [-87 -88 50]; B4 = [-92 -93 50]; B5 = [-164 165 50];		S1	0	0	0	0
			S2	0	60.1683	0	0.16832
			S3	85.1631	90.2682	0	0.3140

	B6 = [-102 102 50];						
21	B1 = [56 -57 50]; % B2 = [-104 105 50]; B3 = [-89 -90 50]; B4 = [-95 -96 50]; B5 = [-160 161 50]; B6 = [-111 112 50];		S1	0	0	0	0
			S2	0	75.5283	0	15.5283
			S3	75.3919	113.3146	0	25.2168
22	B1 = [80 -67 50]; % B2 = [-105 -106 50]; B3 = [-100 -100 50]; B4 = [97 -96 50]; B5 = [161 169 50]; B6 = [-130 126 50];		S1	0	0	0	0
			S2	0	60.9313	0	0.9313
			S3	84.7039	91.3903	0	1.4215
23	B1 = [80 -67 50]; % B2 = [-105 -106 50]; B3 = [-100 -100 50]; B4 = [98 -96 50]; B5 = [160 169 50]; B6 = [-130 126 50];		S1	0	0	0	0
			S2	0	61.0311	0	1.0311
			S3	84.6636	91.5288	0	1.5654
24	B1 = [80 -67 50]; % B2 = [-110 -106 50]; B3 = [-105 -100 50]; B4 = [98 -96 50]; B5 = [160 169 50]; B6 = [-130 126 50];		S1	0	0	0	0
			S2	0	61.1086	0	1.1086
			S3	84.6368	91.6623	0	1.7015
25	B1 = [80 -67 50]; % B2 = [-110 -106 50]; B3 = [-105 -100 50]; B4 = [98 -96 50]; B5 = [150 169 50]; B6 = [-120 126 50];		S1	0	0	0	0
			S2	0	61.0154	0	1.0154
			S3	84.6841	91.5225	0	1.55497
26	B1 = [80 -67 50]; % B2 = [-100 -106 50]; B3 = [-105 -100 50]; B4 = [97 -96 50]; B5 = [130 169 50]; B6 = [-120 126 50];		S1	0	0	0	0
			S2	0	60.1836	0	0.1836
			S3	85.1204	90.2575	0	0.2842

27	B1 = [85 -77 50]; % B2 = [-100 -106 50]; B3 = [-105 -100 50]; B4 = [97 -96 50]; B5 = [130 160 50]; B6 = [-120 126 50];		S1	0	0	0	0
			S2	0	60.1775	0	0.1775
			S3	85.1091	90.2551	0	0.27756

28	B1 = [85 -77 50]; % B2 = [-105 -106 50]; B3 = [-105 -100 50]; B4 = [97 -96 50]; B5 = [110 160 50]; B6 = [-120 126 50];		S1	0	0	0	0
			S2	0	60.0974	0	0.0974
			S3	85.1481	90.1341	0	0.1998
29	B1 = [85 -77 50]; % B2 = [-105 -106 50]; B3 = [-105 -100 50]; B4 = [100 -96 50]; B5 = [110 160 50]; B6 = [-150 120 50];		S1	0	0	0	0
			S2	0	60.0670	0	0.0670
			S3	85.1211	90.1099	0	0.1636
30	B1 = [85 -75 50]; % B2 = [-105 -110 50]; B3 = [-105 -100 50]; B4 = [100 -96 50]; B5 = [110 160 50]; B6 = [-150 120 50];		S1	0	0	0	0
			S2	0	60.0461	0	0.04614
			S3	85.1204271414	90.08342133792	0	0.1464
31	B1 = [85 -75 50]; % B2 = [-110 -110 50]; B3 = [-105 -100 50]; B4 = [140 -96 50]; B5 = [110 160 50]; B6 = [-150 120 50];		S1	0	0	0	0
			S2	0	60.0362	0	0.03624
			S3	85.1297	90.0834	0	0.1542

32	B1 = [50 -75 50]; % B2 = [-72 -110 50]; B3 = [-105 -100 50]; B4 = [90 -96 50]; B5 = [110 160 50]; B6 = [-150 120 50];		S1	0	0	0	0
			S2	0	60.2388		0.2388
			S3	85.1258	90.3293		0.3526
33	B1 = [50 -75 50]; % B2 = [-72 -110 50]; B3 = [-105 -100 50]; B4 = [95 -96 50]; B5 = [110 160 50]; B6 = [-135 120 50];		S1	0	0	0	0
			S2	0	60.2366	0	0.2366
			S3	85.1236	90.3265	0	0.3491
34	B1 = [50 75 50]; % B2 = [-72 -110 50]; B3 = [-105 -110 50]; B4 = [95 -96 50]; B5 = [110 160 50]; B6 = [-135 120 50];		S1	0	0	0	0
			S2	0	60.3760	0	0.3760
			S3	85.1162	90.4844	0	0.4982
35	B1 = [55 75 50]; % B2 = [-72 -110 50]; B3 = [-105 -110 50]; B4 = [95 -96 50]; B5 = [110 150 50]; B6 = [-135 120 50];		S1	0	0	0	0
			S2	0	60.3741	0	0.3741
			S3	85.0906	90.4919	0	0.5002
36	B1 = [102 161 50]; % B2 = [-111 -129 50]; B3 = [108 -119 50]; B4 = [-104 -154 50]; B5 = [106 -195 50]; B6 = [179 -197 50];		S1	0	0	0	0
			S2	0	60.1331	0	0.1331
			S3	85.1194	90.1892	0	0.2237
37	B1 = [57 75 50]; % B2 = [-77 -110 50]; B3 = [-115 -110 50]; B4 = [95 -96 50]; B5 = [110 150 50];		S1	0	0	0	0
			S2	0	60.3658	0	0.3658
			S3	85.0919	90.4819	0	0.4906

	B6 = [-135 120 50];						
38	B1 = [57 75 50]; % B2 = [-77 -110 50]; B3 = [-115 -110 50]; B4 = [95 -96 50]; B5 = [115 150 50]; B6 = [-155 125 50];		S1	0	0	0	0
			S2	0	60.35597526272	0	0.35595
			S3	85.0706	90.4779	0	0.4831
39	B1 = [57 75 50]; % B2 = [-70 -110 50]; B3 = [-116 -110 50]; B4 = [96 -96 50]; B5 = [115 150 50]; B6 = [-155 125 50];		S1	0	0	0	0
			S2	0	60.3578	0	0.3578
			S3	85.0705	90.4805	0	0.4856
40	B1 = [57 75 50]; % B2 = [-70 -110 50]; B3 = [-116 -110 50]; B4 = [90 -96 50]; B5 = [117 150 50]; B6 = [-135 125 50];		S1	0	0	0	0
			S2	0	60.3589	0	0.3589
			S3	85.0670	90.4830		0.4876
41	B1 = [57 75 50]; % B2 = [-77 -110 50]; B3 = [-116 -110 50]; B4 = [92 -96 50]; B5 = [117 150 50]; B6 = [-135 125 50];		S1	0	0	0	0
			S2	0	60.3568	0	0.3568
			S3	85.0674	90.4802	0	0.4849
42	B1 = [57 75 50]; % B2 = [-78 -110 50]; B3 = [-118 -110 50]; B4 = [93 -96 50]; B5 = [117 150 50]; B6 = [-135 125 50];		S1	0	0	0	0
			S2	0	60.3559	0	0.3559
			S3	85.0676	90.4790	0	0.4837
43	B1 = [57 75 50]; % B2 = [-78 -110 50]; B3 = [-118 -110 50];		S1	0	0	0	0
			S2	0	60.3556	0	0.3556
			S3	85.071	90.4771	0	0.48249740742

	B4 = [94 -96 50]; B5 = [115 150 50]; B6 = [-127 125 50];			7			6524
44	B1 = [57 75 50]; %		S1	0	0	0	0
	B2 = [-78 -110 50];		S2	0	60.3500	0	0.3500
	B3 = [-118 -110 50]; B4 = [94 -92 50]; B5 = [115 150 50]; B6 = [-127 125 50];		S3	85.0706	90.4643	0	0.4696
45	B1 = [86 75 50]; %		S1	0	0	0	0
	B2 = [-90 -110 50];		S2	0	60.2751	0	0.2751
	B3 = [-118 -110 50]; B4 = [95 -92 50]; B5 = [115 150 50]; B6 = [-127 125 50];		S3	85.0877	90.3780	0	0.3880
46	B1 = [86 75 50]; %		S1	0	0	0	0
	B2 = [-92 -110 50];		S2	0	60.2774	0	0.2774
	B3 = [-108 -110 50]; B4 = [95 -92 50]; B5 = [115 150 50]; B6 = [-127 125 50];		S3	85.0870	90.3809	0	0.3907
47	B1 = [86 75 50]; %		S1	0	0	0	0
	B2 = [-92 -110 50];		S2	0	60.2778	0	0.27782
	B3 = [-108 -110 50]; B4 = [95 -93 50]; B5 = [115 150 50]; B6 = [-127 125 50];		S3	85.0872	90.3819	0	0.3918
48	B1 = [86 75 50]; %		S1	0	0	0	0
	B2 = [-92 -110 50];		S2	0	60.2781	0	0.2781
	B3 = [-108 -110 50]; B4 = [95 -93 50]; B5 = [116 150 50]; B6 = [-125 125 50];		S3	85.0850	90.3830	0	0.3923
49	B1 = [86 75 50]; %		S1	0	0	0	0
	B2 = [-92 -110 50];		S2	0	60.2555	0	0.2555
	B3 = [-82 -110 50];		S3	85.205	90.3175	0	0.2555

	B4 = [95 -93 50]; B5 = [60 150 50]; B6 = [-125 125 50];			1			
50	B1 = [86 75 50]; % B2 = [-92 -110 50]; B3 = [-85 -110 50]; B4 = [95 -93 50]; B5 = [60 150 50]; B6 = [-125 125 50];		S1	0	0	0	0
			S2	0	60.2546	0	0.2546
			S3	85.2048	90.3164	0	0.3770

Code

```
% WITH ERROR

% H -> Depth
H1 = 50; % Depth of S1
H2 = 60; % Depth of S2
H3 = 70; % Depth of S3

% Depth Difference from H1
DH2 = H1-H2;
DH3 = H1-H3;

% Beacon Nodes
B1 = [ 150   90   H1];
B2 = [-100 -120   H1];
B3 = [ -80  130   H1];
B4 = [ 140  -70   H1];
B5 = [  60  120   H1];
B6 = [ -90 -130   H1];

S1 = [0 0 0]; % Sensor Coordinate
S2 = [0 60 0];
S3 = [85 90 0];

% from 1st Beacon Node
d14 = pdist2(B1,S1,'euclidean');
d24 = pdist2(B1,S2,'euclidean');
d34 = pdist2(B1,S3,'euclidean');

DistBTWSenosrsAndB1 = [d14 d24 d34];

% from 2nd Beacon Node
d15 = pdist2(B2,S1,'euclidean');
d25 = pdist2(B2,S2,'euclidean');
d35 = pdist2(B2,S3,'euclidean');

% from 3rd Beacon Node
d16 = pdist2(B3,S1,'euclidean');
d26 = pdist2(B3,S2,'euclidean');
d36 = pdist2(B3,S3,'euclidean');

% from 4th Beacon Node
d17 = pdist2(B4,S1,'euclidean');
d27 = pdist2(B4,S2,'euclidean');
d37 = pdist2(B4,S3,'euclidean');

% from 5th Beacon Node
```

```
d18 = pdist2(B5,S1,'euclidean');
d28 = pdist2(B5,S2,'euclidean');
d38 = pdist2(B5,S3,'euclidean');

% from 6th Beacon Node
d19 = pdist2(B6,S1,'euclidean');
d29 = pdist2(B6,S2,'euclidean');
d39 = pdist2(B6,S3,'euclidean');

%%%%%%%%%%%%%%%% Adding Gaussian Error %%%%%%%%%%%%%%%%

% 1st Beacon Node
d14 = d14+erf(d14);
d24 = d24+erf(d24);
d34 = d34+erf(d34);

ErroredDistBTWSenosrsAndB1 = [d14 d24 d34];

% 2nd Beacon Node
d15 = d15+erf(d15);
d25 = d25+erf(d25);
d35 = d35+erf(d35);

% 3rd Beacon Node
d16 = d16+erf(d16);
d26 = d26+erf(d26);
d36 = d36+erf(d36);

% 4th Beacon Node
d17 = d17+erf(d17);
d27 = d27+erf(d27);
d37 = d37+erf(d37);

% 5th Beacon Node
d18 = d18+erf(d18);
d28 = d28+erf(d28);
d38 = d38+erf(d38);

% 6th Beacon Node
d19 = d19+erf(d19);
d29 = d29+erf(d29);
d39 = d39+erf(d39);


% From Cayley - Menger Determinant
a11=d14^2;  a12=d24^2;  a13=d34^2;  a14=-(d14^2-d34^2)*(d24^2-
d14^2);  a15=-(d24^2-d14^2)*(d34^2-d24^2);  a16=1;  b1=(d24^2-
d34^2)*(d34^2-d14^2);
a21=d15^2;  a22=d25^2;  a23=d35^2;  a24=-(d15^2-d35^2)*(d25^2-
d15^2);  a25=-(d25^2-d15^2)*(d35^2-d25^2);  a26=1;  b2=(d25^2-
d35^2)*(d35^2-d15^2);
```

```
a31=d16^2;  a32=d26^2;  a33=d36^2;  a34=-(d16^2-d36^2)*(d26^2-
d16^2);  a35=-(d26^2-d16^2)*(d36^2-d26^2);  a36=1;  b3=(d26^2-
d36^2)*(d36^2-d16^2);
a41=d17^2;  a42=d27^2;  a43=d37^2;  a44=-(d17^2-d37^2)*(d27^2-
d17^2);  a45=-(d27^2-d17^2)*(d37^2-d27^2);  a46=1;  b4=(d27^2-
d37^2)*(d37^2-d17^2);
a51=d18^2;  a52=d28^2;  a53=d38^2;  a54=-(d18^2-d38^2)*(d28^2-
d18^2);  a55=-(d28^2-d18^2)*(d38^2-d28^2);  a56=1;  b5=(d28^2-
d38^2)*(d38^2-d18^2);
a61=d19^2;  a62=d29^2;  a63=d39^2;  a64=-(d19^2-d39^2)*(d29^2-
d19^2);  a65=-(d29^2-d19^2)*(d39^2-d29^2);  a66=1;  b6=(d29^2-
d39^2)*(d39^2-d19^2);

% Augmented Matrix
A = [a11 a12 a13 a14 a15 a16
     a21 a22 a23 a24 a25 a26
     a31 a32 a33 a34 a35 a36
     a41 a42 a43 a44 a45 a46
     a51 a52 a53 a54 a55 a56
     a61 a62 a63 a64 a65 a66];

% Just for confirmation. Never Used
Matrix = A;

% cond = cond(A);

% Result
B = [b1
     b2
     b3
     b4
     b5
     b6];

% mldivide function
x = A\B;
% x = pinv(A)*B;

%%%%%%%%%%%%%%%%%%%% Unknown Inner Distances %%%%%%%%%%%%%%%%%
% Distance Between S1 - S2
d12 = sqrt(x(3)/(1-x(4)-x(5)));
% Distance Between S1 - S3
d13 = sqrt((x(3)*x(5))/(1-x(4)-x(5)));
% Distance Between S2 - S3
d23 = sqrt((x(3)*x(4))/(1-x(4)-x(5)));

% Just for confirmation. Never Used
DistBTWSensors = [d12 d13 d23];

%%%%%%%%%%%%%%%%%%  Final  Coordinates  Respect  of  S1(0,0,0)
%%%%%%%%%%%%%%%%%%%%%
```

```
y2 = d12;
y3 = (d12^2+d13^2-d23^2)/(2*d12);
x3 = sqrt(d13^2-((d12^2+d13^2-d23^2)/(2*d12))^2);

% Just for Representation
S  = [ 0  0 0
       0 y2 0
      x3 y3 0];

 % Part: 2 : Coordinates respect to the Beacon Node

 % Distence Between Projected Beacon P4 and Sensors From
Pythagorean Theoram
 D14 = sqrt(d14^2-H1^2);
 D24 = sqrt(d24^2-H1^2);
 D34 = sqrt(d34^2-H1^2);

% Just for Representation. Never Used
 DistBTWSensorsP4 = [D14 D24 D34];

% Coordinates of Beacon
% x4 = 0.5*d12* sqrt(4d12*D14^2-(D14^2-D24^2+d12^2)^2);
 y4 = (D14^2 - D24^2 + d12^2)/(2*d12);
 x4 = sqrt(D14^2 - y4^2);

 % Projected Coordinates of Beacon -> P4
 P4 = [x4 y4 0];
 % Coordinates of Beacon B1
 z4=H1;
 S4 = [x4 y4 z4];

 % After Cartecian Transformation Sensors Coordinate with
respect to Beacon

 fS4 = [0 0 0];              % Concidering Beacon positaion on
Origin
 fS1 = [-x4 -y4 -z4];         % Coordinates of S1 respect of
Beacon
 fS2 = [-x4 (y2-y4) -z4];    % Coordinates of S2 respect of
Beacon
 fS3 = [x3-x4 (y3-y4) -z4]; % Coordinates of S3 respect of
Beacon

 % Just for Representation.
 FinalSensorsCoordinates = [ fS4
                             fS1
                             fS2
                             fS3];
```

```
% Distance between 1st Beacon Node and Calculated Sensors Coordinates
Cald14 = pdist2(fS4,fS1,'euclidean');
Cald24 = pdist2(fS4,fS2,'euclidean');
Cald34 = pdist2(fS4,fS3,'euclidean');

% Just for Representation.
CalDistBTWSensorsAndB1 = [Cald14 Cald24 Cald34];



% After Adding Depth
 FS1 = [-x4 -y4 -z4];               % Coordinates of S1 respect of Beacon
 FS2 = [-x4   (y2-y4) -z4+DH2];    % Coordinates of S2 respect of Beacon
 FS3 = [x3-x4 (y3-y4) -z4+DH3];    % Coordinates of S3 respect of Beacon

% Just For Representation.
CoordinatesOfSensors = [ fS4
                         FS1
                         FS2
                         FS3 ];

x1 = 0;
y1 = 0;
z1 = 0;
x2 = 0;
z2 = 0;
z3 = 0;
  %%%%%%%%%%%%%%%%%%%%%%%%%%
  fx1 = -x4;
  fy1 = -y4;
  fz1 = -z4;
  fx2 = -x4;
  fy2 = y2-y4;
  fz2 = -z4+DH2;
  fx3 = x3-x4;
  fy3 = y3-y4;
  fz3 = -z4+DH3;
figure,
scatter3(150 , 90, 50,'d','filled');
hold on, scatter3(-100 , -120, 50,'d','filled');
hold on, scatter3(-80 , 130, 50,'d','filled');
hold on, scatter3(140 , -70, 50,'d','filled');
hold on, scatter3(60 , 120, 50,'d','filled');
hold on, scatter3(-90 , -120, 50,'d','filled');
hold on, scatter3(x1,y1,z1,'*');
hold on; scatter3(x2,y2,z2,'*');
hold on; scatter3(x3,y3,z3,'*');
```

```
hold on, scatter3(fx1,fy1,fz1,'o');
hold on; scatter3(fx2,fy2,fz2,'o');
hold on; scatter3(fx3,fy3,fz3,'o');
```