Department of Electronics and Telecommunications Engineering

Research Project
On

Performance Evaluation Of Two-Hop Wireless Link Under Rayleigh Fading

Prepared By:

Shusoma Haque
2012-1-55-003

Masrur Arefin Ratul
2012-2-55-040

Supervised By:

Professor Dr. M. Ruhul Amin
Department of ECE
East West University

Professor Dr. Md. Imdadul Islam
Department of ECE
East West University

August 2016

Declaration

This is certified that the project is done by us under the course “Project (ETE-498)”. The project of **Performance Evaluation of Two-Hop Wireless Link** has not been submitted elsewhere for the requirement of any degree or any other purpose except for publication.

Shusoma.Haque
ID: 2012-1-55-003

Masrur Arefin Ratul
ID: 2012-2-55-040

Acceptance

This project paper is submitted to the **Department of Electronics and Communications Engineering**, **East West University** id submitted in partial fulfillment of the requirements for the degree of **B.Sc** in **ETE** under complete supervision of the undersigned.

Professor Dr. M. Ruhul Amin
Department of ECE
East West University

Professor Dr. Md. Imdadul Islam
Department of ECE
East West University

ACKNOWLEDGEMENT

The completion of our project work brings with it a sense of satisfaction, but it is never complete without thanking everyone who made it possible and whose constant support has crowned our efforts with success.

First of all, we would like to express our sincere gratitude to our advisor **Prof. Dr. M. Ruhul Amin** and **Dr. Md. Imdadul Islam** for the continuous support throughout the course of this project effort. Their guidance helped us all the time. Without their initial effort this project would not have been undertaken. We have successfully executed the goal of the project due to their continuous and patient mentoring throughout the entire process. We are extremely grateful to them as they give us the opportunities and exposure that we could never had if we did not worked with them.

In addition, we would like to thank all the faculty members of ECE department for their guidance and support towards completing our B.Sc. Degree and also this project work. We also like to thank the authors mentioned in the reference page, without their work we would be nowhere.

We are thankful to our honorable parents for their great support and inspiration until the end of this project. We would also like to thanks all our friends and batch-mates for supporting us and encouraging us. The name without whom our thanksgiving is incomplete is the almighty Allah, Who allowed us to reach here today.

Thank you all for your great support to us.

Signature:

…………………………………. ……………………………………..

Shusoma Haque
ID: 2012-1-55-003
Department of ECE
East West University

Masrur Arefin Ratul
ID: 2012-2-55-040
Department of ECE
East West University

ABSTRACT

Now-a-days instead of direct communication between Transmitter (T) and Receiver (R), the relayed link is used to enhance SNR. Statistical model of instantaneous SNR is derived hence the profile of cdf and pdf instantaneous SNR is plotted. In this project work we provided analytical model of two-hop wireless link under Rayleigh fading environment to evaluate outage probability and symbol error rate (SER). We compared the performance of dual-hop link for SISO, MISO, SIMO, MIMO system against the variation of SNR.

CONTENTS

List of Figures

CHAPTER 1

INTRODUCTION

In wireless communication various types of the signal variation occur. This variation occurs because of multipath fading, path loss and mobility. As the order of modulation increases the propagation error and bit error rate also increases [1]. To improve wireless communication links reliability MIMO technology is used. In MIMO Orthogonal Space-Time Block Coding (OSTBC) is used. The specialty of OSTBC is that it can provide channel state information at the transmitter (CSIT) without complex feedback link [2]. Another technology is also used to improve reliability of wireless link which is known as relayed transmission technique. To improve the reliability of wireless link, between transmitter and receiver a relay is placed. The function of relay is to divide the link in two parts. This is called two hop wireless links. If it divides the link in more than two parts then it is called multiple hop wireless links [3].

Two-hop transmission technologies have attracted much interest in wireless communications due to its connectivity when direct transmission is impractical because of possible shadowing effects or path loss attenuation In real wireless communication environments, the links between cooperating nodes can experience asymmetric fading conditions. Several works have analyzed two-hop transmission systems which use Amplify and Forward (AF) or Decode and Forward (DF) technique operating over fading channels[4].Relaying technique is used for extending the coverage area (possibly shadowed) in wireless environments.

Consider a communication system in which two terminals are communicating via a third terminal that acts as a relay.

Relay is the repeater, node or channel that inter connects the source (transmitter) and destination (receiver), through which they can receive and transmission formation from source to destination, when the direct link between the source and destination is in deep fade, so the signals to the destination propagate through two or more hops or links in series. Relay is used to boost and resend a signal to the next terminal. Relay transmission has been identified as one of the core technologies that could enable high speed information transfer over challenging wireless environment. Delay has made a major part in mobile communication and wireless broadband access. This scenario was encountered originally in bent pipe satellites where the primary function of the spacecraft is to relay the uplink carrier into a downlink. It is also common in various fixed microwave links by enabling greater coverage without the need of large power at the transmitter. More recently, this concept has gained new actuality in collaborative/cooperative wireless communication systems. Few statistical models are used to describe fading in wireless environments in communication system analysis. The most frequently used distributions are Rayleigh, Rician, Nakagami-m, Nakagami-q and Wiebull [5, 6].

Two hop transmission system can be classified into two main categories, depending on the nature and complexity of the relays, 1) Regenerative relay systems and 2) Non-regenerative relay systems. In the case of regenerative also known as Decode and forward systems, the relay fully decodes the signal that went through the first hop and retransmits the decoded version into the second hop. Non regenerative also known as amplify and forward systems use less complex relays that just amplify and forward the incoming signal without performing any sort of decoding [7].

Cooperative communication is a hot topic of current research and many people believe it to be the next big step after multiple-input multiple-output (MIMO) systems. The basic idea is that multiple nodes cooperate in order to increase the link quality, reliability and data rate of the system. In the future, the density of active nodes competing for a common wireless channel in cellular as well as access or ad-hoc networks will increase significantly. Therefore, node cooperation is an efficient means of achieving these gains [8]. An overview of several cooperative diversity protocols for wireless networks can for example be found in. One source transmits to one destination while a single relay assists. The capacity of the general relay channel is still not known. Moreover, there is even no cooperation strategy known that works best for this general case. At least two different basic signal processing strategies are distinguished at the relay. The first strategy -known as decode-and forward (DF) protocol – involves decoding of the source transmission at the relay. The re-encoded and possibly compressed signal is then forwarded to the destination. In terms of sum rate this protocol is close to optimal when the source-relay channel is excellent, which is practically the case when source and relay are physically close or when dedicated relays are placed intentionally in a way that a good connection to the source is ensured [9].

For the second strategy - known as compress-and-forward (CF) or quantize-and-forward protocol – there lay does not decode the source signal, but uses its observations in a different way. The received signals quantized (and possibly compressed) and then forwarded to the destination. This protocol is most efficient in cases where the source-relay and the source-destination channels are of comparable quality, and the relay-destination link is good. In this situation, the relay may not be able to decode the source signal, but nevertheless has an independent signal observation that can be used to assist the decoding at the destination. Finally, the amplify-and-forward (AF) protocol is a special case of the CF strategy, where the signal processing at the relay is only linear. For a multi-antenna relay this means that each transmit antenna may forward a linear combination of the received signals of all receive antennas [10].

The entire project work is organized as: Chapter 2 provides mathematical analysis of MIMO wireless channel, Chapter 3 provides theory of two-hop wireless link along with derivation of probability density function of combined SNR, Chapter 4 provides the theoretical model, Chapter 5 provides result based analysis of chapter 2 and chapter 3, finally chapter 6 concludes the entire analysis of the project work.

CHAPTER 2

THEORY OF MULTIPLE INPUT MULTIPLE OUTPUT

2.1 Multiple-Input, Multiple-Output Antenna Systems

2.1.1 Introduction

Multiple-input multiple-output (MIMO) wireless communications include space diversity on receive as a special case. Most important, however, are the following three points:

a. The fading phenomenon is viewed not as a nuisance, but rather as an environmental source of possible enrichment.
b. Space diversity at both the transmitting and receiving ends of the wireless communications link provides the basis for a significant increase in channel capacity or spectral efficiency.
c. Unlike increasing capacity with conventional techniques, increasing channel capacity with MIMO is achieved by increasing computational complexity while maintaining the primary communication resources (i.e., total transmit power and channel bandwidth) fixed.

2.1.2 Co-antenna Interference

Fig. 2.1 shows the block diagram of a MIMO wireless link [18]. The signals transmitted by the N_t transmit antennas over the wireless channel are all chosen to lie inside a common frequency band. Naturally, the transmitted signals are scattered differently by the channel. Moreover, due to multiple signal transmissions, the system experiences a spatial form of signal-dependent interference referred to as co-antenna interference (CAI).

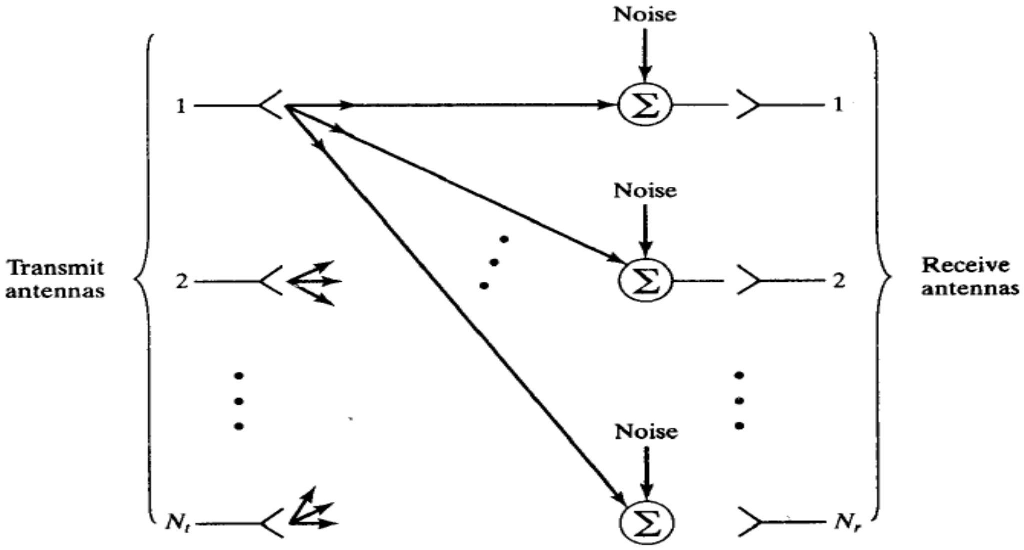

Fig. 2.1: Block diagram of MIMO wireless link with N_t transmit antennas and N_r receive antennas.

2.1.3 Basic Baseband Channel Model

Let us consider a MIMO narrowband wireless communication system built around a flat-fading channel and with N_t transmit antennas and N_r receive antennas [18]. The antenna configuration is hereafter referred to as the pair (N_t, N_r). For a statistical analysis of the MIMO system in what follows, we use baseband representations of the transmitted and received signals, as well as of the channel. In particular, we introduce the following notation:

- The spatial parameter

$$N = \min\{N_t, N_r\} \tag{2.1}$$

Defines a new degree of freedom introduced into the wireless communication system by using a MIMO channel with N_t transmit antennas and N_r receive antennas.

- The $N_t \times 1$ vector

$$\mathbf{s}(n) = \left[\tilde{s}_1(n), \tilde{s}_2(n), \cdots, \tilde{s}_{N_t}(n)\right]^T \tag{2.2}$$

Denotes the complex signal vector transmitted by the N_t antennas at discrete time *n*. The symbols constituting the vector $\mathbf{s}(n)$ are assumed to have zero mean and common variance σ_s^2.

The total transmit power is fixed at the value

$$P = N_t \sigma_s^2. \tag{2.3}$$

For P to be maintained constant, the variance σ_s^2 (i.e., the power radiated by each transmit antenna) must be inversely proportional to N_t.

- For the flat-fading, and therefore, memory less, channel, we may use $\tilde{h}_{ik}(n)$ to denote the sampled complex gain of the channel from transmit antenna *k* to receive antenna *i* at discrete time *n*, where $i = 1, 2, \cdots, N_r$ and $k = 1, 2, \cdots, N_t$. We may thus express the $N_r \times N_t$ complex channel matrix as

$$H(n) = \begin{bmatrix} \tilde{h}_{11}(n) & \tilde{h}_{12}(n) & \cdots & \tilde{h}_{1N_t}(n) \\ \tilde{h}_{21}(n) & \tilde{h}_{22}(n) & \cdots & \tilde{h}_{2N_t}(n) \\ \cdots & \cdots & \cdots & \cdots \\ \tilde{h}_{N_r 1}(n) & \tilde{h}_{N_r 2}(n) & \cdots & \tilde{h}_{N_r N_t}(n) \end{bmatrix}. \tag{2.4}$$

- The system of equations

$$\tilde{x}_i(n) = \sum_{k=1}^{N_t} \tilde{h}_{ik}(n)\,\tilde{s}_k(n) + \tilde{w}_i(n), \qquad \begin{matrix} i = 1, 2, \cdots, N_r \\ k = 1, 2, \cdots, N_t \end{matrix} \tag{2.5}$$

Defines the complex signal received at the *i*-th antenna due to the transmitted symbol $\tilde{s}_k(n)$ radiated by the *k*-th antenna. The term $\tilde{w}_i(n)$ denotes the additive complex channel noise perturbing $\tilde{x}_i(n)$.

Let the $N_r \times 1$ vector

$$\mathbf{x}(n) = \left[\tilde{x}_1(n), \tilde{x}_2(n), \cdots, \tilde{x}_{N_r}(n)\right]^T \quad (2.6)$$

denote the complex received signal vector and the $N_r \times 1$ vector

$$\mathbf{w}(n) = \left[\tilde{w}_1(n), \tilde{w}_2(n), \cdots, \tilde{w}_{N_r}(n)\right]^T \quad (2.7)$$

denote the complex channel noise vector. We may then rewrite the system of equations, Eq. (2.5) in the compact matrix form as

$$\mathbf{x}(n) = \mathbf{H}(n)\mathbf{s}(n) + \mathbf{w}(n). \quad (2.8)$$

Eq. (2.8) describes the basic complex-channel model for MIMO wireless communications, assuming the use of a flat-fading channel. The equation describes the input-output behavior of the channel at discrete time *n*. To simplify the exposition, hereafter we suppress the dependence on time *n* by writing

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{w}, \quad (2.9)$$

where it is understood that all four vector-matrix terms of the equation, namely, **s**, **H**, **w**, and **x**, are in fact dependent on the discrete time *n*. Fig. 2.2 depicts the basic channel model of Eq. (2.9).

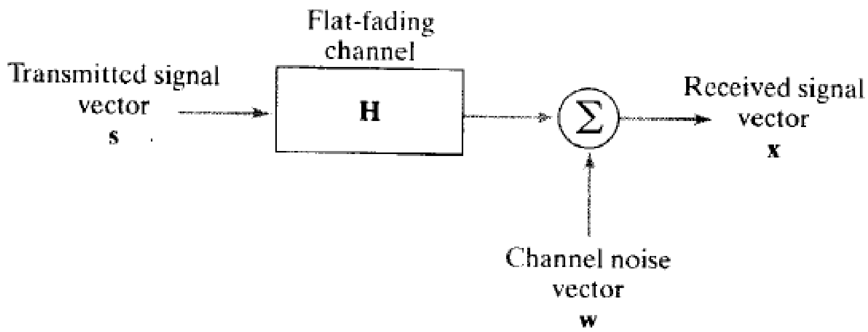

Fig. 2.2: Depiction of the basic channel model of Eq. (2.9).

For mathematical tractability, we assume a Gaussian channel model made up of three elements relating to the transmitter, channel, and receiver, respectively [18]:

1. The N_t symbols constituting the transmitted signal vector s are drawn from a white complex Gaussian code book; that is, the symbols $\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_{N_t}$ are independently and identically distributed (iid) complex Gaussian random variables with zero mean and common variance $\sigma_{\mathbf{s}}^2$.

   Hence, the correlation matrix of the transmitted signal vector s is defined by

$$\begin{aligned} \mathbf{R_s} &= \mathbf{E}(\mathbf{s}\mathbf{s}^{\dagger}) \\ &= \sigma_{\mathbf{s}}^2 \mathbf{I}_{N_t}, \end{aligned} \tag{2.10}$$

   where $\mathbf{I}_{N_t}$ is the $N_t \times N_t$ identity matrix.

2. The $N_r \times N_t$ elements of the channel matrix H are drawn from an ensemble of iid complex random variables with zero mean and unit variance, as shown by the complex distribution

$$h_{ik} : \quad N(0,1/\sqrt{2}) + jN(0,1/\sqrt{2}), \quad \begin{matrix} i = 1, 2, \cdots, N_r \\ k = 1, 2, \cdots, N_t \end{matrix} \tag{2.11}$$

   Where N denotes a real Gaussian distribution. On this basis, we find that the amplitude component h_{ik} is Rayleigh distributed, so we sometimes speak of the MIMO channel as a rich Rayleigh scattering environment. By the same token, we also find that the squared amplitude component, namely, $|h_{ik}|^2$, is a chi-square random variable with mean

$$\mathbf{E}\left(|h_{ik}|^2\right) = 1 \quad \text{for all } i \text{ and } k. \tag{2.12}$$

3. The N_r elements of the channel noise vector w are iid complex Gaussian random variables with zero mean and common variance $\sigma_{\mathbf{w}}^2$; that is, the correlation matrix of the noise vector w is given by

$$\begin{aligned} \mathbf{R_w} &= \mathbf{E}(\mathbf{w}\mathbf{w}^{\dagger}) \\ &= \sigma_{\mathbf{w}}^2 \mathbf{I}_{N_r}, \end{aligned} \tag{2.13}$$

where $\mathbf{I}_{N_r}$ is the $N_r \times N_r$ identity matrix.

In light of Eq. (2.3) and the assumption that h_{ik} is a normalized random variable with zero mean and unit variance, the average signal-to-noise ratio (SNR) at each receiver input is given by

$$\begin{aligned} \rho &= \frac{P}{\sigma_{\mathbf{w}}^2} \\ &= \frac{N_t\, \sigma_{\mathbf{s}}^2}{\sigma_{\mathbf{w}}^2}, \end{aligned} \tag{2.14}$$

which, for a prescribed noise variance $\sigma_{\mathbf{w}}^2$, is fixed once the total transit power P is fixed. We note also that (1) all the N_t transmitted signals occupy a common channel bandwidth and (2) the SNR at each branch ρ, is independent of N_r.

The idealized Gaussian model described herein is applicable to indoor local area networks and other wireless environments where the mobility of the user's terminals is limited. The model, however, ignores the unavoidable ambient noise, which, as a result of experimental measurements, is known to be decidedly non-Gaussian due to the impulse nature of human-made electromagnetic interference as well as natural noise.

2.2 MIMO Capacity for Channel Known at the Receiver

With the basic complex channel model at hand, we are now ready to focus the discussion on the primary issue of interest: the channel capacity of a MIMO wireless link. Two cases will be considered. The first case considers a link that is stationary and therefore ergodic. The second case considers a nonergodic link, assuming quasi-stationary from one data burst to another [18].

2.2.1 Ergodic Capacity

It is to be noted that the information capacity of a real additive white Gaussian noise (AWGN) channel, subject to the constraint of a fixed transmit power *P*, is defined by [18]:

$$C = B\log_2\left(1+\frac{P}{\sigma_{\mathbf{w}}^2}\right), \tag{2.15}$$

where *B* is the channel bandwidth and $\sigma_{\mathbf{w}}^2$ is the noise variance measured over *B*.

Given a time-invariant channel, Eq. (2.15) defines the maximum data rate that can be transmitted over the channel with an arbitrarily small probability of error incurred as a result of the transmission. With the channel used *K* times for the transmission of *K* symbols in, say, *T* seconds, the transmission capacity per unit time is (*T*/*K*) times the formula for *C* given in Eq. (2.15). Recognizing that $K = 2BT$, in accordance with the sampling theorem, we may express the information capacity of the AWGN channel in the equivalent form:

$$C = \frac{1}{2}\log_2\left(1+\frac{P}{\sigma_{\mathbf{w}}^2}\right) \quad \text{bits/s/Hz.} \tag{2.16}$$

We note that 1 bit per second per hertz corresponds to 1 bit per transmission.

With wireless communications as the medium of interest, let us consider next the case of a complex, flat-fading channel with the receiver having perfect knowledge of the channel state. The capacity of such a channel is given by

$$C = \mathbf{E}\left[\log_2\left(1+\frac{|h|^2 P}{\sigma_{\mathbf{w}}^2}\right)\right] \quad \text{bits/s/Hz,} \tag{2.17}$$

where the expectation operator **E** is taken over the gain $h(n)$ of the channel, and the channel is assumed to be stationary and ergodic. In recognition of this assumption, *C* is commonly referred to as the ergodic capacity of the flat-fading channel, and the channel coding is applied across fading intervals (i.e., over an “ergodic” interval of channel variation with time).

It is important to note that the scaling factor ½ is missing from the capacity formula of Eq. (2.17). The reason for this omission is the fact that equation refers to a complex baseband channel, whereas Eq. (2.16) refers to a real channel. The fading channel covered by Eq. (2.17) operates on a complex signal – a signal with in-phase and quadrature components. Therefore, such a complex channel is equivalent to two real channels with equal capacities and operating in parallel – hence the result presented in that equation.

Eq. (2.17) applies to the simple case of a *single-input, single-output* (SISO) flat-fading channel. Generalizing this formula to the case of a *multiple-input, multiple-output* (MIMO) flat-fading channel, governed by the Gaussian model described ion Section 2.1.2, we find that the ergodic capacity of the MIMO channel is given by [Appendix 5.A, Eq. (A27)]

$$C = \mathbf{E}\left[\log_2\left\{\frac{\det\left(\mathbf{R_w} + \mathbf{H}\mathbf{R_s}\mathbf{H}^\dagger\right)}{\det\left(\mathbf{R_w}\right)}\right\}\right] \quad \text{bits/s/Hz,} \qquad (2.18)$$

which is subject to the constraint

$$\max_{\mathbf{R_s}} tr\left(\mathbf{R_s}\right) \le P,$$

where P is the constant transmit power. The expectation in Eq. (2.18) is over the random channel matrix $\mathbf{H}$, and the superscript † denotes Hermitian transposition; $\mathbf{R_s}$ and $\mathbf{R_w}$ are, respectively, the correlation matrices of the transmitted signal vector $\mathbf{s}$ and channel noise vector $\mathbf{w}$. A detailed derivation of Eq. (2.18) is given in the next section.

In general, it is difficult to evaluate Eq. (2.18), except for the Gaussian model in Section 2.3. In particular, substituting Eqs. (2.10) and (2.13) into Eq. (2.18) and simplifying yields

$$C = \mathbf{E}\left[\log_2\left\{\mathbf{I}_{N_r} + \frac{\sigma_s^2\mathbf{H}\mathbf{H}^\dagger}{\sigma_{\mathbf{w}}^2}\right\}\right] \quad \text{bits/s/Hz.} \qquad (2.19)$$

Invoking the definition of the average SNR ρ introduced in Eq. (2.14), we may rewrite Eq. (2.19) in the following equivalent form

$$C = \mathbf{E}\left[\log_2 \left\{ \det\left(\mathbf{I}_{N_r} + \frac{\rho}{N_t} \mathbf{H}\mathbf{H}^{\dagger} \right) \right\} \right] \quad \text{bits/s/Hz.} \qquad (2.20)$$

Eq. (2.20), defining the ergodic capacity of a MIMO flat-fading channel, involves the determinant of an $N_r \times N_r$ sum matrix followed by the logarithm to the base 2. Accordingly, this formula of Eq. (2.20) is sometimes known as *log-det capacity formula* for a Gaussian MIMO channel.

It is to be mentioned here that the formula of Eq. (2.20) assumes that $N_t \geq N_r$ for the $N_r \times N_r$ matrix product $\mathbf{HH}^{\dagger}$ to be of full rank. For the alternate case, when $N_t \leq N_r$, which makes the $N_t \times N_t$ matrix product $\mathbf{H}^{\dagger}\mathbf{H}$ to be of full rank, then the log-det capacity formula takes the form

$$C = \mathbf{E}\left[\log_2 \left\{ \det\left(\mathbf{I}_{N_t} + \frac{\rho}{N_r} \mathbf{H}^{\dagger}\mathbf{H} \right) \right\} \right] \quad \text{bits/s/Hz,} \qquad (2.21)$$

whereas before, the expectation is over the channel matrix **H**.

We note that Eqs. (2.20) and (2.21) are equivalent, in that either one of them applies to all $\{N_t, N_r\}$ antenna configurations. The two formulas differentiate themselves only when the full-rank issue is of concern.

Clearly, Eq. (2.17), pertaining to a conventional flat-fading link with a single antenna at both ends, is a special case of the log-det capacity formula. Specifically, for $N_t = N_r = 1$ (i.e., no spatial diversity), $\rho = P / \sigma_{\mathbf{w}}^2$, and $\mathbf{H} = [h]$ (with dependence on discrete time *n*), Eq. (2.19) reduces to Eq. (2.17).

Another insightful result that follows from the log-det capacity formula is that if $N_t = N_r = N$, then as *N* approaches infinity, the capacity *C* defined in Eq. (2.19) grows asymptotically (at least) linearly with *N*.

That is,

$$\lim_{N \to \infty} \frac{C}{N} \geq \text{ constant.} \tag{2.22}$$

The asymptotic formula of Eq. (2.22) may be stated in words as follows:

> *The ergodic capacity of a MIMO flat-fading wireless link with an equal number N of transmit and receive antennas grows roughly proportionately with N.*

What this statement teaches us is that, by increasing the computational complexity resulting from the use of multiple antennas at both the transmitting and receiving ends of a wireless link, we are able to increase the spectral efficiency of the link in a far greater manner than is possible by conventional means (e.g., increasing the transmit SNR). The potential for this very sizable increase in the spectral efficiency of a MIMO wireless communication system is attributed to the key parameter $N = \min\{N_t, N_r\}$, which defines the number of spatial degrees of freedom provided by the system.

2.2.2 Two Other Special Cases of the Log-Det Formula: Capacities of Receive and

Transmit Diversity Links

Naturally, the log-det capacity formula of Eq. (2.20) for the channel capacity of an $\{N_t, N_r\}$ wireless link includes the channel capacities of receive and transmit diversity links as special cases [18]:

1. Diversity on Receive Channel. The log-det capacity formula of Eq. (2.21) applies to this case. Specifically, for $N_t = 1$, the channel matrix $\mathbf{H}$ reduces to a column vector, and with it, Eq. (2.21) reduces to

$$C = \mathbf{E}\left[\log_2\left\{\det\left(1 + \rho \sum_{i=1}^{N_r} |h_i|^2\right)\right\}\right] \quad \text{bits/s/Hz.} \tag{2.23}$$

Compared with the channel capacity of Eq. (2.17), for a single input, single output fading channel with $\rho = P/\sigma_{\mathbf{w}}^2$, the squared channel gain $|h|^2$ is replaced by the sum of squared amplitudes $|h_i|^2, \quad i = 1, 2, \cdots, N_r$. Eq. (2.23) expresses the ergodic capacity due to the linear combination of the receive-antenna outputs, which is designed to maximize the information contained in the N_r received signals about the transmitted signal. This is simply a restatement of the maximal-ratio combining principle.

2. Diversity on Transmit Channel. The log-det capacity formula of Eq. (2.20) applies to this second case. Specifically, for $N_r = 1$, the channel matrix H reduces to a row vector, and with it, Eq. (2.20) reduces to

$$C = \mathbf{E}\left[\log_2\left\{\det\left(1 + \frac{\rho}{N_t}\sum_{k=1}^{N_t}|h_k|^2\right)\right\}\right] \quad \text{bits/s/Hz,} \qquad (2.24)$$

where the matrix product $\mathbf{HH}^{\dagger}$ is replaced by the sum of squared amplitudes $|h_k|^2, \quad k = 1, 2, \cdots, N_t$. Compared with Case 1 on receive diversity, the capacity of the diversity on transmit channel is reduced because the total transmit power is held constant, independently of the number N_t transmit antennas.

2.2.3 Outage Capacity

To realize the log-det capacity formula of Eq. (2.20) [18], the MIMO channel code needs to see an ergodic process of the random-channel processes, which, in turn, results in a hardening of the rate of reliable transmission to the $\mathbf{E}[\log_2\{\det(\cdot)\}]$ information rate (i.e., the channel capacity approaches the log-det formula). As in all information-theoretic arguments, the bit error rate would go to zero asymptotically in the block length of the code, thereby entailing a long transportation delay from the sender to the sink. In practice, however, the MIMO wireless channel is often non-ergodic, and the requirement is to operate the channel under delay constraints. The issue of interest is then summed up as follows:

How much information can be transmitted across a non- ergodic channel, particularly if the channel code is long enough to see just one random-channel matrix?

In the situation described here, the rate of reliable information transformation (i.e., the strict Shannon-sense capacity) is zero, since, for any positive rate, the probability that the channel would not support such a rate is nonzero.

To get around this serious conceptual difficulty, the notion of outage is introduced into the characterization of the MIMO link. Specifically,

The outage probability of a MIMO link is defined as the probability for which the link is in a state of outage (i. e., failure) for data transmitted across the link at a certain rate R, measured in bits per second per hertz.

To proceed on this probabilistic basis, it is customary to operate the MIMO link by transmitting data in the form of bursts or frames, invoking a quasi-static model governed by four points:

1. The burst is long enough to accommodate the transmission of a large number of symbols, which, in turn, permits the use of an idealized infinite-time horizon that is basic to information theory.
2. Yet the burst is short enough that the wireless link can be treated as quasi static during each burst; the slow variation is used to justify the assumption that the receiver can acquire perfect knowledge of the channel state.
3. The channel matrix is permitted to change, say, from burst k to the next burst, k+1, thereby accounting for statistical variations of the link.
4. The different realizations of the transmitted signal vector $\mathbf{s}$ are drawn from a white Gaussian codebook: that is, the correlation matrix of $\mathbf{s}$ is defined by Eq. (2.10).

Points 1 and 4 pertain to signal transmission, while points 2 and 3 pertain to the channel.

To proceed with the evaluation of the outage probability, we first note that points 1 trough 4 of the stochastic model just described for a non stationary wireless link permit us to build on some of the results discussed in Section 2.2.1. In particular, in light of the log-det capacity formula of Eq. (2.20), we may view the random variable

$$C_k = \mathbf{E}\left[\log_2\left\{\det\left(\mathbf{I}_{N_r} + \frac{\rho}{N_t}\mathbf{H}_k\mathbf{H}_k^{\dagger}\right)\right\}\right] \quad \text{bits/s/Hz for burst } k \tag{2.25}$$

as the expression for a sample of the wireless link. In other words, with the random-channel matrix $\mathbf{H}_k$ varying from one burst to the next, C_k will itself vary in a corresponding way. A consequence of this random behavior is that, occasionally, a draw from the cumulative distribution function of the wireless link results in a value for C_k that is inadequate to support reliable communication over the link, in which case the link is said to be in an outage state. Correspondingly, for a given transmission strategy, we define the outage probability at rate R as

$$P_{outage}(R) = \text{Prob}\{C_k < R \text{ for some burst } k\} \tag{2.26}$$

or, equivalently,

$$P_{outage}(R) = \text{Prob}\left\{\log_2\left\{\det\left(\mathbf{I}_{N_r} + \frac{\rho}{N_t}\mathbf{H}_k\mathbf{H}_k^{\dagger}\right)\right\} < R \text{ for some burst } k\right\} \tag{2.27}$$

On this basis, we may define the outage capacity of the MIMO link as the *maximum bit rate that can be maintained across the link for all bursts of data transmissions (i. e., all possible channel states) for a prescribed outage probability.*

2.3 Channel Known at the Transmitter

The log-det capacity formula of Eq. (2.20) is based on the premise that the transmitter has no knowledge of the channel state. Knowledge of the channel state, however, can be made available to the transmitter by first estimating the channel matrix $\mathbf{H}$ at the receiver and then sending this estimate to the transmitter via a feedback channel. In such a scenario, the capacity is optimized over the correlation matrix of the transmitted signal vector **s**, subject to the power constraint; that is, the trace of this correlation matrix is less than or equal to the constant transmit power P.

From a practical perspective, it is important to note that the capacity gain provided by knowledge of the channel state at the transmitter over the log-det formula of Eq.
(2.20) is significant only at low SNRs; the gain reduces to zero as the SNR increases [18].

2.4 Singular-Value Decomposition of the Channel Matrix

We may gain further insight into the behavior of a MIMO wireless communication system by applying what is known as the singular-value decomposition (SVD) to the channel matrix of the system. He relationship between this algebraic decomposition of a rectangular matrix and the eigende composition of a Hermitian matrix formed by multiplying the matrix by its Hermitian transpose is discussed in Appendix 5.C [18].

To begin the exposition, let us consider the matrix $\mathbf{HH}^{\dagger}$ in the log-det capacity formula of Eq. (2.20). This product satisfies the Hermitian property for all **H**. We may therefore diagonalize $\mathbf{HH}^{\dagger}$ by invoking the eigende composition of Hermitian matrix and so we write

$$\mathbf{U}^{\dagger}\mathbf{HH}^{\dagger}\mathbf{U} = \mathbf{\Lambda}, \tag{2.28}$$

where the two new matrices **U** and $\mathbf{\Lambda}$ are described as follows:

- The matrix $\mathbf{\Lambda}$ is a diagonal matrix whose N_r elements are the eigenvalues of the matrix product $\mathbf{HH}^{\dagger}$.
- The matrix **U** is a unitary matrix whose N_r columns are the eigenvectors associated with the eigenvalues of $\mathbf{HH}^{\dagger}$.

By definition, the inverse of a unitary matrix is equal to the Hermitian transpose of the matrix, as shown by

$$\mathbf{U}^{-1} = \mathbf{U}^{\dagger} \tag{2.29}$$

or, equivalently,

$$\mathbf{U}\mathbf{U}^{\dagger} = \mathbf{U}^{\dagger}\mathbf{U} = \mathbf{I}_{N_r}\,, \tag{2.30}$$

where $\mathbf{I}_{N_r}$ is the $N_r \times N_r$ identity matrix.

Let $N_t \times N_t$ matrix **V** be another unitary matrix; that is,

$$\mathbf{V}\mathbf{V}^{\dagger} = \mathbf{V}^{\dagger}\mathbf{V} = \mathbf{I}_{N_t}\,, \tag{2.31}$$

where $\mathbf{I}_{N_t}$ is the $N_t \times N_t$ identity matrix. Since the multiplication of a matrix by the identity matrix leaves the matrix unchanged, we may inject the matrix product $\mathbf{V}\mathbf{V}^{\dagger}$ into the center of the left-hand side of Eq. (2.28), thus:

$$\mathbf{U}^{\dagger}\mathbf{H}\left(\mathbf{V}\mathbf{V}^{\dagger}\right)\mathbf{H}^{\dagger}\mathbf{U} = \mathbf{\Lambda}\,. \tag{2.32}$$

The left-hand side of Eq. (2.32), representing a square matrix, is recognized as the product of two rectangular matrices: the $N_r \times N_t$ matrix $\mathbf{U}^{\dagger}\mathbf{H}\mathbf{V}$ and the $N_t \times N_r$ matrix $\mathbf{V}^{\dagger}\mathbf{H}^{\dagger}$, which are the Hermitian of each other. Let the $N_t \times N_t$ matrix **D** denote a new diagonal matrix related to the $N_r \times N_r$ diagonal matrix $\mathbf{\Lambda}$ with $N_r \le N_t$ by

$$\mathbf{\Lambda} = [\mathbf{D} \quad \mathbf{0}][\mathbf{D} \quad \mathbf{0}]^{\dagger}\,, \tag{2.33}$$

where the null matrix **0** is added to maintain proper overall matrix dimensionality of the equation. Except for some zero elements, **D** is the square root of $\mathbf{\Lambda}$. Then, examining Eqs. (2.32) and (2.33) and comparing terms, we deduce the new decomposition

$$\mathbf{U}^{\dagger}\mathbf{H}\mathbf{V} = [\mathbf{D} \quad \mathbf{0}]. \tag{2.34}$$

Eq. (2.34) is a mathematical statement of the singular-value decomposition (SDV) theorem, according to which we have the following descriptions.

- The elements of the diagonal matrix

$$\mathbf{D} = \mathrm{diag}\left(d_1, d_2, \cdots, d_{N_t}\right) \tag{2.35}$$

  are the singular values of the channel matrix **H**.

- The columns of the unitary matrix

$$\mathbf{U} = (\mathbf{u}_1, \quad \mathbf{u}_2, \quad \cdots, \quad \mathbf{u}_{N_r}) \tag{2.36}$$

  are the left singular vectors of matrix **H**.

- The columns of the unitary matrix

$$\mathbf{V} = (\mathbf{v}_1, \quad \mathbf{v}_2, \quad \cdots, \quad \mathbf{v}_{N_t}) \tag{2.37}$$

  are the right singular vectors of matrix **H**.

Applying the singular-value decomposition of Eq. (2.34) to the basic channel model of Eq. (2.9), show that for $N_r \le N_t$:

$$\bar{\mathbf{x}} = [\mathbf{D}, \mathbf{0}]\bar{\mathbf{s}} + \overline{\mathbf{w}}, \tag{2.38}$$

where

$$\bar{\mathbf{x}} = \mathbf{U}^{\dagger}\mathbf{x}, \tag{2.39}$$

$$\bar{\mathbf{s}} = \mathbf{V}^{\dagger}\mathbf{s}, \tag{2.40}$$

and

$$\overline{\mathbf{w}} = \mathbf{U}^{\dagger}\mathbf{w}. \tag{2.41}$$

Using the definitions of Eqs. (2.35) through (2.37), we may rewrite the decomposed channel model of Eq. (2.38) in the scalar form

$$\bar{x}_i = d_i \bar{s}_i + \overline{w}_i, \qquad i = 1, 2, \cdots, N_r., \tag{2.42}$$

According to Eq. (2.42), singular-value decomposition of the channel matrix **H** has transformed to MIMO wireless link with $N_r \le N_t$ into a N_r virtual channels, as illustrated in Fig. 2.3. We note that $\bar{s}_i = 0$ for $N_r < i \le N_t$. The virtual channels are all decoupled from each other in that they constitute a parallel set of N_r single-input, single-output (SISO) channels, with each channel being described by the scalar input-output relation of Eq. (2.42). A comparison of the channel models of Figs. 2.2 and 2.3 immediately reveals the decoupling facilitated in the virtual model of Fig. 2.3 by the singular-value decomposition of the channel matrix **H**.

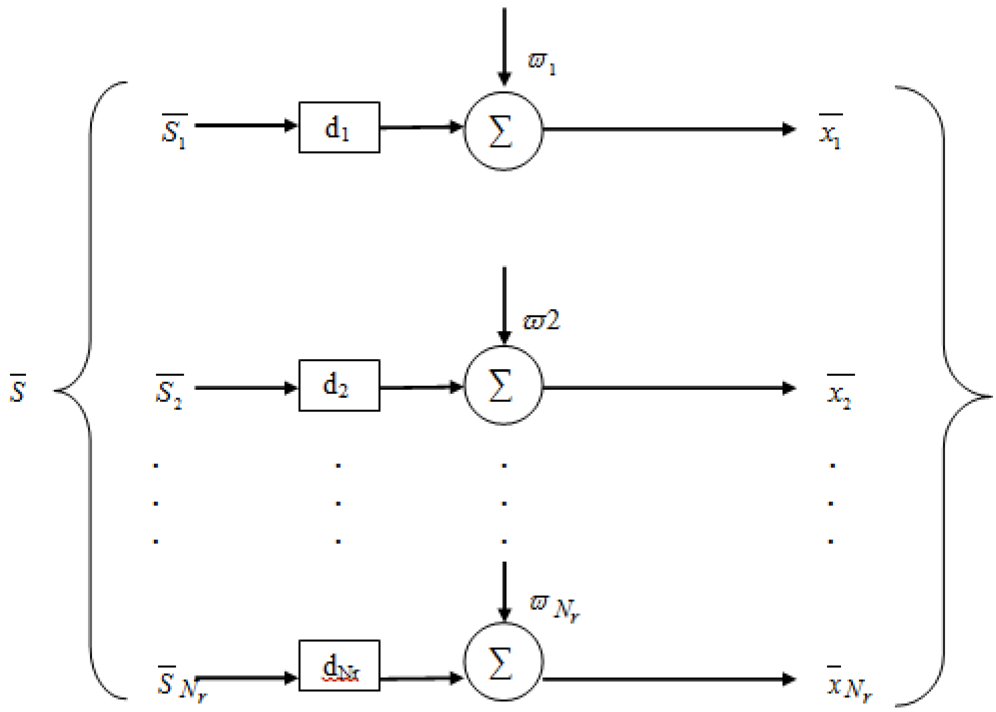

Fig. 2.3: Set of N_r virtual secoupled channels resulting from the singular-value decomposition of the channel matrix H, assuming that $N_t \leq N_r$.

2.4.1 Eigende composition of the Log-det capacity Formula

The log-det formula of Eq. (2.20) for the ergodic capacity of a MIMO link involves the matrix product $\mathbf{HH}^\dagger$. Substituting Eq. (2.20) into Eq. (2.28) leads to the spectral decomposition of $\mathbf{HH}^\dagger$ in terms of N_r eigen modes, with each eigen mode corresponding to virtual data transmission using a pair of right- and left-singular vectors of the channel matrix **H** as the transmit and receive antenna weights, respectively. Thus, we may write [18]:

$$\begin{aligned} \mathbf{HH}^\dagger &= \mathbf{U}\Lambda\mathbf{U}^\dagger \\ &= \sum_{i=1}^{N_r} \lambda_i \mathbf{u}_i \mathbf{u}_i^\dagger , \end{aligned} \tag{2.43}$$

where the outer product $\mathbf{u}_i\mathbf{u}_i^\dagger$ is an $N_r \times N_r$ matrix with a rank equal to unity. Moreover, substituting the first line of this decomposition into the determinant part of Eq. (2.20) yields

$$\det\left(\mathbf{I}_{N_r} + \frac{\rho}{N_t}\mathbf{H}\mathbf{H}^\dagger\right) = \det\left(\mathbf{I}_{N_r} + \frac{\rho}{N_t}\mathbf{U}\mathbf{\Lambda}\mathbf{U}^\dagger\right). \tag{2.44}$$

Next, invoking the determinant identity

$$\det(\mathbf{I} + \mathbf{AB}) = \det(\mathbf{I} + \mathbf{BA}) \tag{2.45}$$

and then using the defining Eq. (2.30), we may rewrite Eq. (2.44) in the equivalent form

$$\begin{aligned}\det\left(\mathbf{I}_{N_r} + \frac{\rho}{N_t}\mathbf{H}\mathbf{H}^\dagger\right) &= \det\left(\mathbf{I}_{N_r} + \frac{\rho}{N_t}\mathbf{U}^\dagger\mathbf{U}\mathbf{\Lambda}\right)\\ &= \det\left(\mathbf{I}_{N_r} + \frac{\rho}{N_t}\mathbf{\Lambda}\right)\\ &= \prod_{i=1}^{N_r}\left(1 + \frac{\rho}{N_t}\lambda_i\right),\end{aligned} \tag{2.46}$$

where λ_i is the *i*-th eigne value of $\mathbf{H}\mathbf{H}^\dagger$. Finally, substituting Eq. (2.46) into Eq. (2.20) yields

$$C = \mathbf{E}\left[\sum_{i=1}^{N_r}\log_2\left(1 + \frac{\rho}{N_t}\lambda_i\right)\right] \quad \text{bits/s/Hz}, \tag{2.47}$$

which is subject to constant transmit power; the expectation is over the eigenvalues of the matrix product $\mathbf{H}\mathbf{H}^\dagger$. Eq. (2.47) shows that, thanks to the properties of the logarithm, the ergodic capacity of a MIMO wireless communication system is the sum of capacities of N_r virtual single-input, single-output channels defined by the spatial eigen modes of the matrix product $\mathbf{H}\mathbf{H}^\dagger$.

According to Eq. (2.47), the channel capacity C attains its maximum value when equal signal-to-noise ratios ρ / N_t are allocated to each virtual channel in Fig. 2.3 (i.e., the N_r eigen modes of the channel matrix **H** are all equally effective). By the same token, the capacity C is minimum when there is a single virtual channel (i.e., all the eigen modes of the channel matrix **H** are zero except for one). The capacities of actual wireless links lie somewhere between these two extremes.

Specifically, as a result of fading correlation encountered in practice, it is possible for there to be a large disparity amongst the eigenvalues of $\mathbf{HH}^{\dagger}$; that is, one or more of the eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_{N_r}$ may be small. Such a disparity has a quite a detrimental effect on the capacity of the wireless link, compared with the maximal condition under which all the eigenvalues of $\mathbf{HH}^{\dagger}$ are equal. A similar effect may also arise in a Rician fading environment when the line-of-sight (LOS) component is quite strong (i.e., the Rician factor is greater than, say, 10 dB), in which case one eigen mode of the channel is dominant. For example, when the angle spread of the incoming radio waves impinging on a linear array of receive antennas is reduced from 60^{O} to 0.6^{O}, the complementary cumulative distribution function (ccdf) of a MIMO wireless communications system with $(N_t, N_r) = (7, 7)$ antenna configuration degenerates effectively to that of a (7, 1) system.

Eq. (2.47), based on the log-det capacity formula of Eq. (2.20), assumes that $N_t \geq N_r$. Using the log-det capacity formula of Eq. (2.21) for the alternative case, $N_t \leq N_r$, and following a procedure similar to that used to derive Eq. (2.47), we may show that

$$C = \mathbf{E}\left[\sum_{i=1}^{N_t} \log_2\left(1 + \frac{\rho}{N_r}\lambda_i\right)\right] \quad \text{bits/s/Hz}, \qquad (2.48)$$

where the expectation is over the λ_i, which denote the eigenvalues of the $N_t \times N_t$ matrix product $\mathbf{H}^{\dagger}\mathbf{H}$.

CHAPTER 3

TWO-HOP WIRELESS LINK

The broadcast nature of wireless transmissions has enabled a new communication paradigm known as "cooperative communications" wherein the source node communicates with the destination node with the help of one or more relay nodes to harness the inherent spatial diversity gain in wireless networks without requiring multiple transceivers at the destination node. It is an active and growing field of research because this form of "user cooperation diversity" has the ability to overcome the practical implementation issue of packing a large number of antenna elements on small-sized hand-held portable wireless devices and sensor nodes, besides enabling the source node to tap into the available resources of local neighboring nodes to increase its throughput, range, reliability, and covertness [4].

Wireless Relaying is a promising solution to overcome the channel impairments and provides high data rate coverage that appears for beyond 3G mobile communications. Dual hop transmission is a technique by which the channel from source to destination is split into two shorter links using a relay. In this case the key idea is that the source relays a signal to destination via a third terminal that acts as a relay. It is an attractive technique when the direct link between the base station and the original mobile terminal is in deep fade or heavy shadowing or there is no direct link between source and destination. On the other hand diversity technique is an effective technique to mitigate the severe form of interference that arises due to the multi path propagation of wireless signal gain without increasing the expenditure of transmission time or bandwidth [11]. Although transmission diversity is clearly advantageous on a cellular base station, it might not be practical for other scenarios. Specially due to cost, size or hardware limitations, a wireless device may not be able to support multiple transmission antennas. In order to overcome this limitation a new form of diversity technique, the cooperative diversity (named so as it comes from user cooperation). It exploits the broadcast nature of the wireless transmission. In cellular, the ad-hoc network when one user is transmitting information to a remote terminal, other users nearby also receive it and transmit the signal to the destination. This process results in multiple copies of same signal from independent fading paths at the destination and brings diversity. Depending on the nature and the complexity of the relays cooperative transmission system can be classified into two main categories; regenerative and non regenerative systems. In regenerative systems, relay fully decodes the signal that went through the first hop. Then retransmit the decoded version to the second hop. This is also referred to as decode- and-forward or digital relaying. On the other hand, non regenerative systems use less complex relays that just amplify and forward the incoming signal without performing any sort of decoding. It is called amplify-and-forward or analog relaying. The performance of both systems has been well studied. In a distributed Space Time Coded (STC) cooperative scheme is proposed, where the relays decode the received symbols from the source and utilize a distributed STC. The number of distributed antennas (distributed relays) for cooperation is generally unknown and also may be not unique. So coordination among the cooperating nodes is needed prior to the use of a specific space time coding scheme. Moreover, if many relay stations transmit signal to destination then it also needs synchronization of carrier phases among several transmit receive pairs which will increase the complexity of receiver as well as cost. Choosing the minimum number of relays for reducing cooperation overhead and saving energy without performance loss is an important concern. There are various protocols proposed to choose the best relay among a collection of available relays in literature. This single relay opportunistic selection provides no performance loss from the perspective of diversity-multiplexing gain trade off, compared to schemes that rely on distributed space time coding [12-14]. Before transmitting signal to destination the best relay is selected based on the instantaneous channel conditions of two hops. This technique can save the transmission power of the network. It also reduces the decoding complexity at receiver side and at the same time achieves diversity gain. However this intermediate relay shall increase the maximum distance between source and destination also increase the spectral efficiency. In this paper in Section 2 we described about MIMO. Section 3 shows some basic properties of relay. Simulation results are also presented in Section 5 and also some results are showed for clearing the concept.

Generally, there are two types of relays: passive and active. Briefly, a passive relay simply reflects an incoming signal to an intended receiver. Passive relays are usually placed at fixed positions such as on a ceiling. As its name implies, passive relays are simply reflectors and thus are not capable of adjusting the direction of reflection to match any changes. Therefore, passive relays are only used in fixed networks; e.g., data centers. Apart from that, the deployment of passive relays introduces high signal attenuation. This is exacerbated by the high path loss of the 60 GHz signal coupled with elongated relayed paths. An active relay functions as a signal repeater and more importantly, it is ableto forward a signal onto any direction using beam forming techniques or directional antennas if needed [9-10].

Recently, researchers have paid significant attention in the field of two-hop wireless links in both the cases of with and without channel state information. To enhance the reliability of a wireless network, relaying techniques are specially used when the link between the sender and receiver are far away or when the direct link is heavily contaminated with noise and fading. Two of the most widely used techniques are amplify-and-forward (AF) relaying and decode-and-forward (DF) relaying. Sometimes, the gain of a relay is kept constant, irrespective of the condition of the wireless channel (fixed gain AF relay). However, sometimes a relay station taking feedback/acknowledgement from the receiver to adjust the gain is called the variable gain AF relay. For the case of partial channel state information (PCSI) is use by the AF technique to estimate the outage probability for a cognitive relay network. The average fading power of a 'source-to-relay' and relay-to-destination (R-D) link are derived in a Rayleigh fading environment. The performance of a multi-hop cooperative relay network where the destination receives a signal from both the relay and sender is derived considering two random variables for signal-to-noise ratio (SNR) where one is from the relay and the other from the sender for k fading. In the AF model is used under the line of sight interference environment. We derived the gain of a relay station in terms of transmitted signal power, the power of interferences, the source-to-relay path gain and noise variance. The idea of a two-hop wireless link is applied in a multi-way relay channel where multiple users exchange information through a relay terminal. The relaying concept is applied in a cognitive radio network using different cooperative protocols like selection amplify-and-forward (SAF), selection decode-and-forward (SDF). Sometimes, among several antennas the best antenna at the transmitting end is selected based on the concept of the water-filling model (to maximize the instantaneous SNR), which is known as transmit antenna selection (TAS). The combination of TAS and MRC under multiple-input multiple-output (MIMO) is analyzed in previous. where the outage probability and bit error rate (BER) of different schemes are compared with the TAS/MRC schemes. The combination of TAS at the transmitter side and SC/MRC at the receiver side is analyzed in with a closed form of solution of the outage probability. A multiple-antenna system is used in two-hop link to improve the reliability of the link (i.e., reducing the ergodic error probability or the outage probability and to enhance the data rate by transmitting a parallel data stream over the antennas. To reduce the ergodic error probability, the SNR needs to be enhanced. The SNR needs narrow beam forming, an increase in the variation of SNRs, and path diversity. The degree of diversity can be improved ina two-hop wireless link by the applying multiple antennas at the transmitter/receiver/relay stations. Several approaches can be imposed in context of MIMO in a two-hop wireless link. For example, one approach is to place multiple antennas at the receiver side by making an appropriate separation between the adjacent antennas depending on the channel state characteristics to get an uncorrelated channel response, which is called, receive diversity. In accordance with the multiple receivers, there are different combining schemes, such as selection combining, equal gain combining, maximum ratio combining (MRC), etc. All of which are used to represent the optimum receiver. The second approach is to place multiple antennas at the transmitter side but a signal that is transmitted simultaneously over several antennas will not get the desired diversity gain. In this case, special coding is required at the transmitter. These special codes are called space-time codes. There are widely used codes like Alamuiti, space-time block codes (OSTBCs) and space-time trellis codes (STTCs) are used to get uncorrelated symbols. The STBC in the previous discussion is also applicable in a single hop wireless link to reduce the BER and to increase the throughput. Sometimes, the best transmitting antenna (TAS) is selected in consideration of the water-filling model. TAS is also used in this paper. Another approach is to use multiple antennas only on the relay station and single antenna on both ends. The reverse model (i.e., multiple antennas) on both ends and a single antenna on the relay are also possible [15].

CHAPTER 4

THE THEORETICAL MODEL

In dual-hop (two-hop) wireless communication, a relay station is used between the

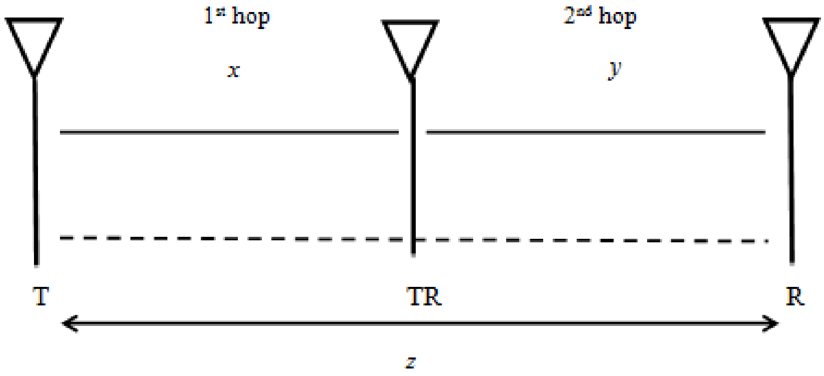

Fig: 4.1(a) Two-hop wireless link.

transmitter and receiver link fig: 4.1(a). The SNR between the transmitter and receiver is x and that of between relay and receiver is y. The end-to-end SNR is expressed as [17],

$$z = \frac{xy}{x + y + c} \tag{4.1}$$

Where the parameter c is real and nonnegative and chosen such as to reflect the configuration of the TR node [18].

4.1 Single input single output system:

For $N_{T=1}$ and N_R=1, instantaneous SNRs of channel path (T-TR), (TR-R) and (T-R),

$$x = x_1 \| h_1 \|^2 \quad (4.2)$$

$$y = y_1 \| h_2 \|^2 \quad (4.3)$$

$$z_d = z_1 \| h_3 \|^2 \quad (4.4)$$

SNR at the receiving end always vary, that's why SNR itself a random variable. To analysis a random variable we use probability density function (PSD). The variations of the SNR at the receiving end sometimes follow Rayleigh distribution, sometimes follow Rician distribution or sometimes follow Nakagami m distribution. In any statistical distribution we always represent the random variable with a average number and a variance.

x_1 is the average SNR of T-TR link, y_1 is the average SNR of TR-R link, z_1 is the average SNR of TR-R link, and h_1, h_2, h_3 are the channel coefficients of the T-TR, TR-R and T-R links respectively.

If we multiply the average SNR with the channel gain we will get the instantaneous channel SNR.

Thus, the equivalent SNR for a SISO system can be written as following [17],

$$z = z_1 \| h_3 \|^2 + \frac{x_1 \| h_1 \|^2 \, . y_1 \| h2 \|^2}{x_1 \| h1 \|^2 + y_1 \| h2 \|^2} \quad (4.5)$$

4.2 Multiple input single output system:

Let's consider a two hop link. Where T represents the transmission link, TR represents the relay and R represents the receiver link. Here we have multiple transmitter antenna and single receiving and relay antenna.

For N_t number of transmission antennas, the SNR of the (T-TR) link is,

$$x = x_1 \sum_{i=1}^{N_t} \| h_{1i} \|^2 \tag{4.6}$$

Here,

x_1 is the average SNR of the link and the value of 'i' is the number of transmitter. h_{1i} represents the channel coefficient for each transmitter (T) to relay (TR) link.

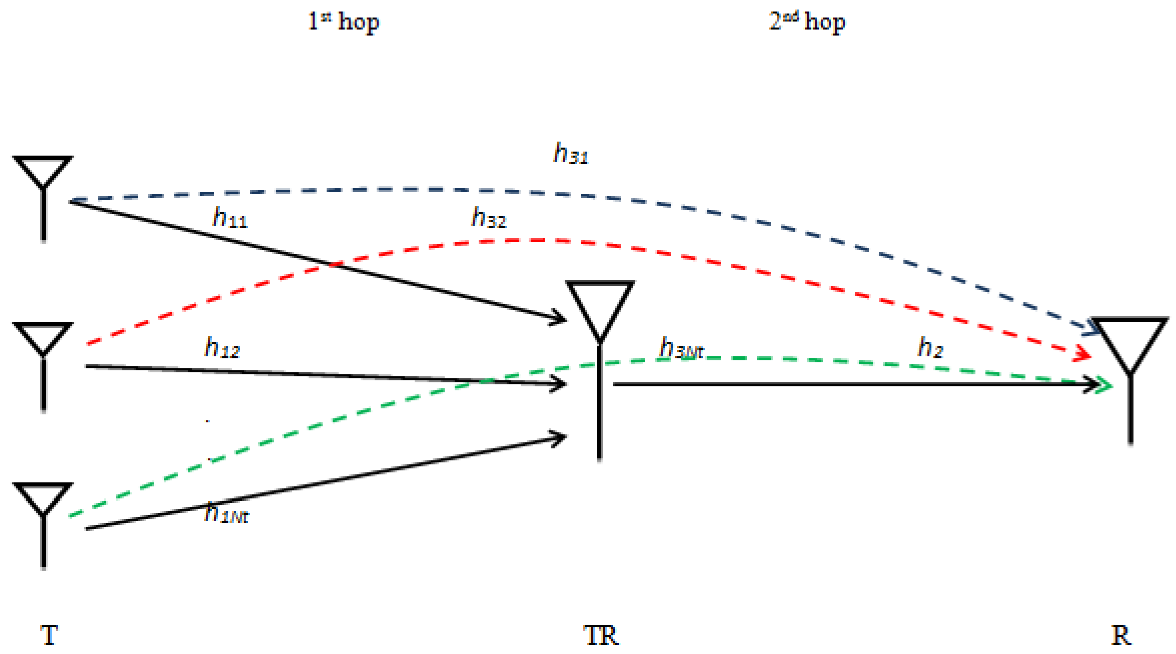

Fig: 4.1(b) Two-hop wireless link (MISO).

For single receiver antenna, SNR of the (TR-T) link is,

$$y = y_1 \| h_2 \|^2 \tag{4.7}$$

Here,
y_1 is the average SNR of the link and h_2 represents the channel coefficient for relay (TR) to receiver (R) link.

For N_t number of transmission and single receiving antenna, the SNR of the direct (T-TR) link is,

$$z_d = z_1 \sum_{i=1}^{N_t} \| h_{3i} \|^2 \tag{4.8}$$

So, the equivalent SNR of MISO system can be written as,

$$z = z_1 \sum_{i=1}^{N_t} \| h_{3i} \|^2 + \frac{x_1 \sum_{i=1}^{N_t} \| h_{1i} \|^2 . y_1 \| h_2 \|^2}{x_1 \sum_{i=1}^{N_t} \| h_{1i} \|^2 + y_1 \| h_2 \|^2} \tag{4.9}$$

4.3 Multiple input multiple output system:

Let's consider a two hop link. Where T represents the transmission link, TR represents the relay and R represents the receiver link. Here we have multiple transmitter and receiving antenna and single relay.

For N_t number of transmission antennas, the SNR of the (T-TR) link is,

$$x = x_1 \sum_{i=1}^{N_t} \| h_{1i} \|^2 \tag{4.10}$$

Here,
x_1 is the average SNR of the link and the value of 'i' is the number of transmitter. h_{1i} represents the channel coefficient for each transmitter (T) to relay (TR) link.

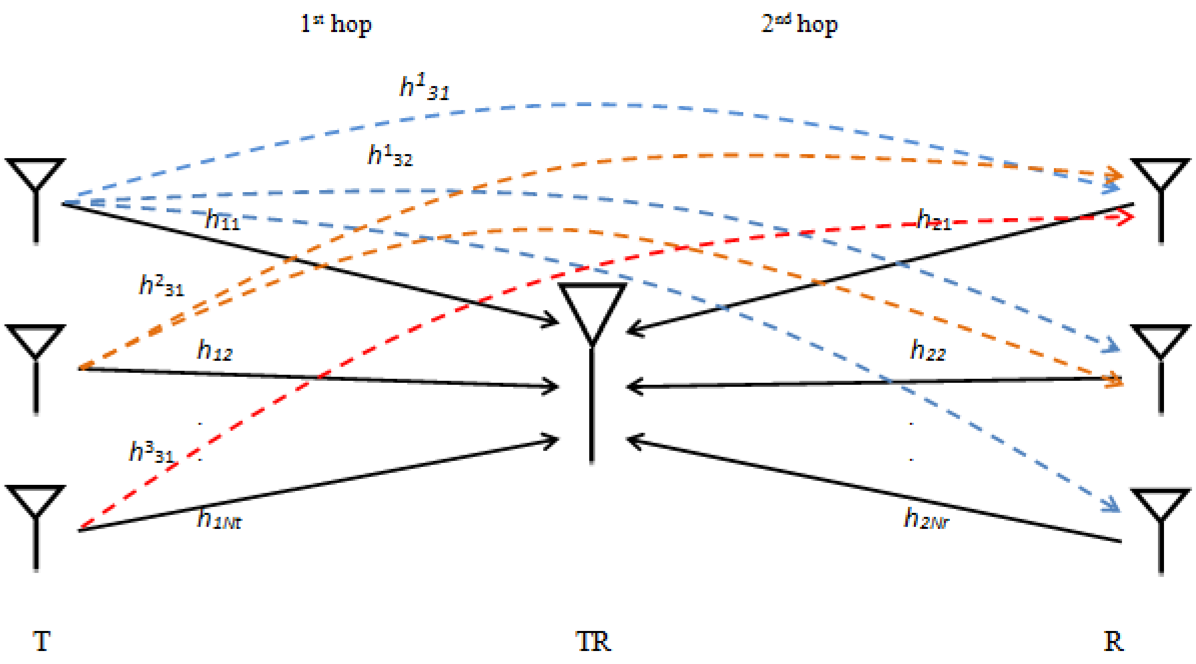

Fig: 4.1(c) Two-hop wireless link (MIMO).

For N_r number of receiver antennas, SNR of the (TR-T) link is,

$$y = y_1 \sum_{j=1}^{N_r} \| h_{2j} \|^2 \tag{4.11}$$

Here,
y_1 is the average SNR of the link and the value of 'j' is the number of receiver. h_{2j} represents the channel coefficient for relay (TR) to each receiver (R) link.

As we know there is a direct link between each transmitter (T) and receiver (R) individually.
If there are N_t numbers of transmission antenna and N_r numbers of receiving antenna, the SNR of the (T-R) link is,

$$z_d = z_1 \sum_{i=1}^{N_t} \sum_{j=1}^{N_r} \| h_{3i}^{j} \|^2 \tag{4.12}$$

Here,
z_1 is the average SNR of the link. The value of '*i*' is the number of transmitter and '*j*' is the number of receiver. h_{3i}^{j} represents the channel coefficient for each transmitter (T) to each receiver (R) link.

So, the equivalent SNR can be written as [17],

$$z = z_1 \sum_{i=1}^{N_t} \sum_{j=1}^{N_r} \| h_{3i}^{j} \|^2 + \frac{x_1 \sum_{i=1}^{N_t} \| h_{1i} \|^2 . y_1 \sum_{j=1}^{N_r} \| h_{2j} \|^2}{x_1 \sum_{i=1}^{N_t} \| h_{1i} \|^2 + y_1 \sum_{j=1}^{N_r} \| h_{2j} \|^2} \qquad (4.13)$$

4.4 Theoretical Formulation:

Let the transmitter, relay and receiver be denoted by T, TR and R respectively, where the instantaneous SNR of link T-TR is x, and TR-R is y and average SNR of link T-TR is, x_{avg} and TR-R is y_{avg}. Equivalent SNR of link T-R is

$$z = \frac{xy}{y+c}. \qquad (4.14)$$

For Rayleigh fading, the probability density function (pdf) is

$$g_X(x) = \frac{1}{x_{avg}} e^{-x/x_{avg}}. \qquad (4.15)$$

The cumulative distribution function (cdf) is

$$\begin{aligned} G_X(x) &= \int_0^x g_X(x) \; dx \\ &= \int_0^x \frac{1}{x_{avg}} e^{-x/x_{avg}} \; dx \qquad (4.16) \\ &= 1 - e^{-x/x_{avg}}. \end{aligned}$$

The cdf of z is

$$
\begin{aligned}
R_Z(\tau) &= \int_0^{\infty} \Pr\left\{ \frac{xy}{y+c} \le \tau \mid y \right\} g_Y(y)\, dy \\
&= \int_0^{\infty} \Pr\left\{ x \le \frac{\tau(y+c)}{y} \mid y \right\} g_Y(y)\, dy \\
&= \int_0^{\infty} G_X \left\{ \frac{\tau(y+c)}{y} \right\} g_Y(y)\, dy \\
&= \int_0^{\infty} \left[1 - e^{-\frac{\tau(\mathrm{y+c})}{y.x_{avg}}} \right] \frac{1}{y_{avg}} e^{-\frac{y}{y_{avg}}} dy.
\end{aligned}
\tag{4.17}
$$

The pdf of z is,

$$
\begin{aligned}
f_Z(\tau) &= \frac{dR_Z(\tau)}{d\tau} \\
&= \left[1 - e^{-\frac{\tau(\mathrm{y+c})}{y.x_{avg}}} \right] \frac{1}{y_{avg}} e^{-\frac{y}{y_{avg}}}.
\end{aligned}
\tag{4.18}
$$

Symbol Error Rate (SER),

$$
SER(y_{av}) = \int_0^{v} r(\tau, y_{av}).0.5.erfc(\beta.\tau) d\tau \tag{4.19}
$$

Outage Probability,

$$
Pout(y_{av}) = \int_0^{v} r(\tau, y_{av}) d\tau \tag{4.20}
$$

For fixed gain relay:

Here $N_t = N_r = n$

$$k_1 = |h_{11}|^2 + |h_{12}|^2 + |h_{13}|^2 + ...|h_{1n}|$$
$$k_2 = |h_{21}|^2 + |h_{22}|^2 + |h_{23}|^2 + ...|h_{21n}|$$

Then the cdf is,

$$\begin{aligned} R_Z(\tau) &= \int_0^{\infty} \Pr\left\{\frac{k_1 x\, k_2 y}{k_2 y + c} \le \tau \,|\, k_2 y\right\} g_y(y)\,dy \\ &= \int_0^{\infty} \Pr\left\{x \le \frac{\tau(k_2 y + c)}{k_1 k_2 y} \,|\, y\right\} g_y(y)\,dy \\ &= \int_0^{\infty} Fx\left\{\frac{\tau(k_2 y + c)}{k_1 k_2 y}\right\} g_y(y)\,dy \end{aligned} \tag{4.21}$$

the pdf is,

$$r_Z(\tau) = \frac{dR_Z(\tau)}{d\tau} \tag{4.22}$$

CHAPTER 5

RESULTS AND DISCUSSIONS

Fig. 5.1 shows the variation in the cdf of equivalent SNR *z*, taking, average value of absolute SNR, x_{av}= 12 and *C* = 1. Fig. 5.2 shows the variation in the pdf of equivalent SNR *z*, Taking, the same numerical values of parameters. The set of curves are down for y_{avg} = 3, 10, and 15.

Fig. 5.3 shows the variation in outage probability taking threshold SNR as a parameter. We get 4 graphs for 4 different threshold SNR and we observe that outage probability increases as the threshold SNR increase. Actually the threshold SNR is the main hurdle of two hop wireless i.e. if the combined SNR is greater than the threshold SNR then the communication is successful, otherwise the receiver will experience huge noise which incurs with huge BER.

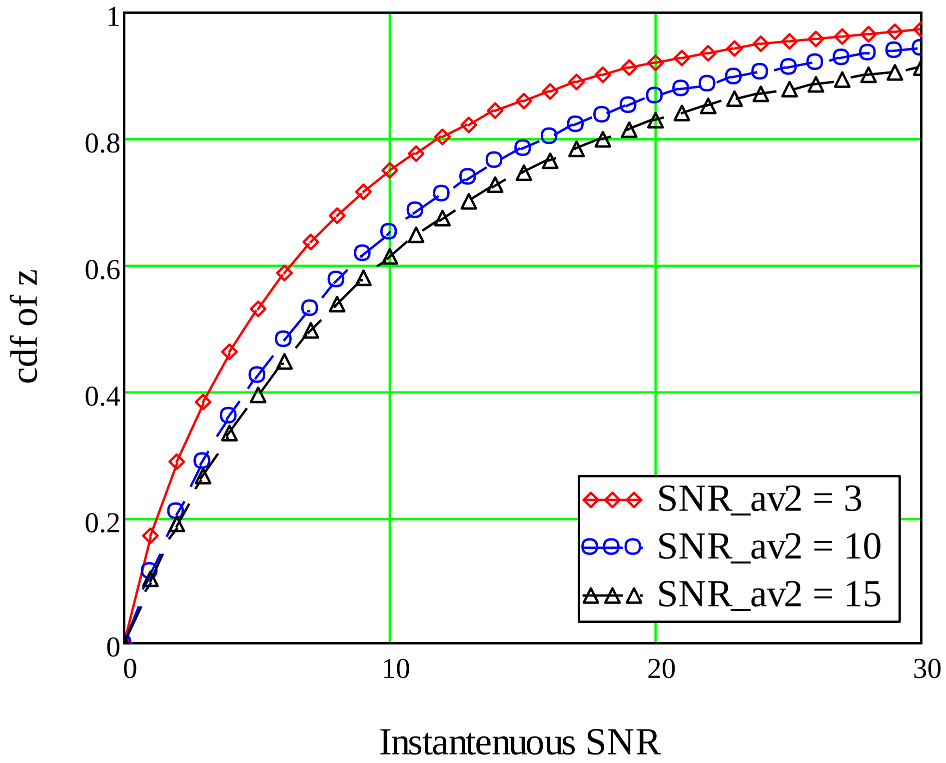

Fig. 5.1: The cdf of equivalent SNR of two-hop link.

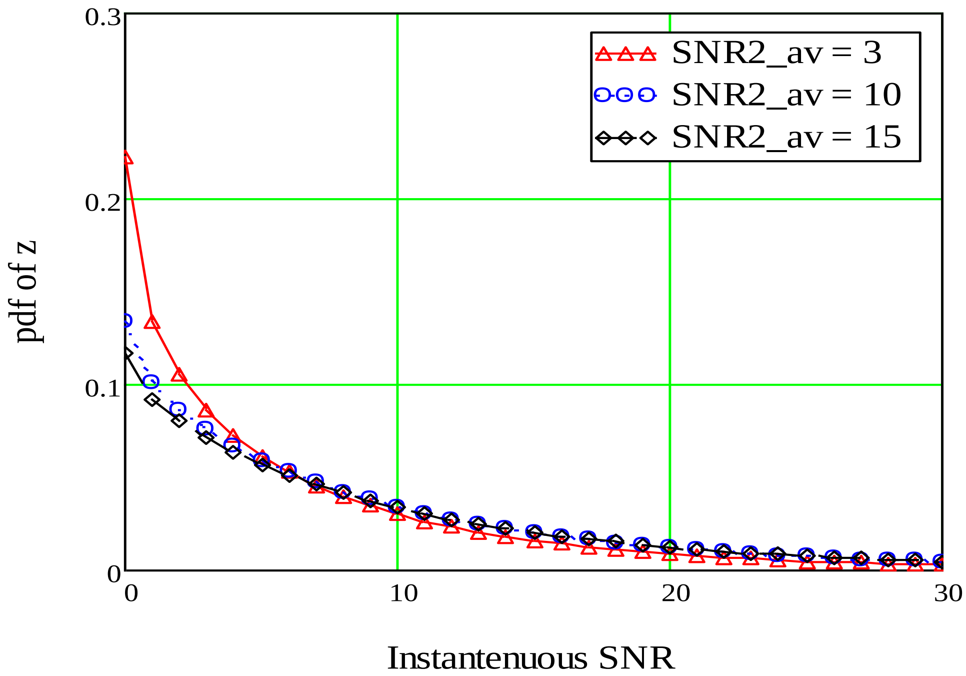

Fig. 5.2: The pdf of equivalent SNR of two-hop link.

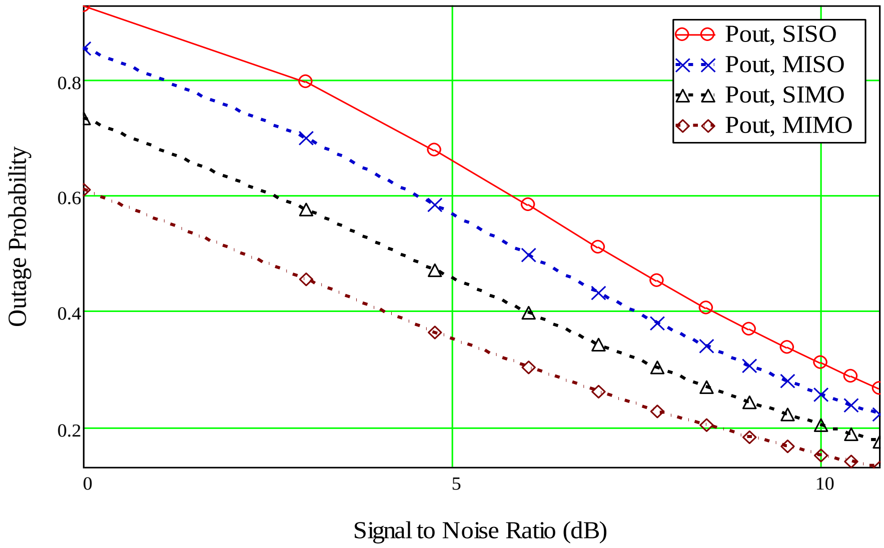

Fig. 5.3: Variation of outage probability against SNR under four schemes of two-hop link.

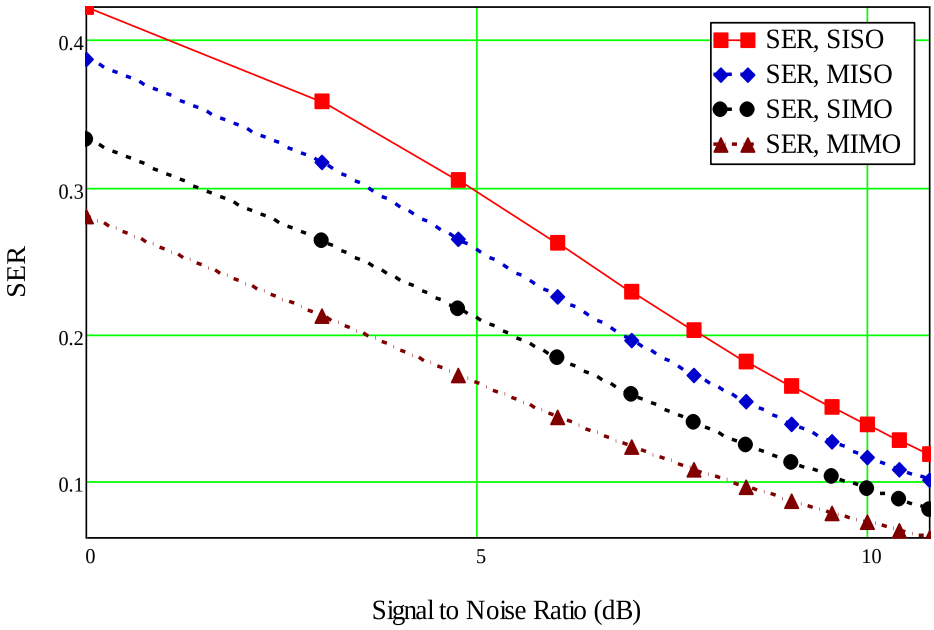

Fig. 5.4: Variation of SER for QPSK.

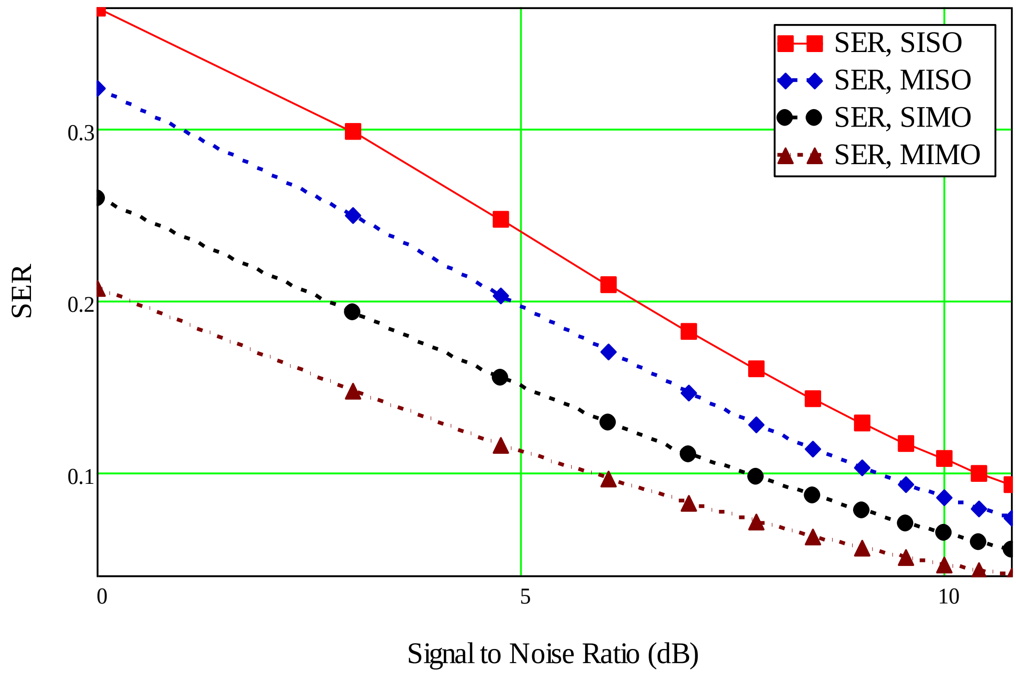

Fig. 5.5: Variation of SER for BPSK.

Here we use four schemes: SISO, MIMO, SIMO and MIMO. From the Fig. 5.3 it is realized that the performance of MIMO is the best and SISO is the worst case because of span diversity. Finally SER of QPSK and BPSK schemes are shown in Fig, 5.4 and 5.5 respectively. Like outage probability, the performance of MIMO is the best, the order of performance are SIMO, MISO, SISO. At the same time SER of QPSK is more than the case of BPSK, which can be explained from signal space.

CHAPTER 6

CONCLUSION

In this project work, we measured the performance of two-hop wireless link in context of outage probability and SER. The best performance is found when MIMO link is used at both transmitting and receiving ends. The performance of SIMO is found better than MISO because of space diversity on weak signal at the receiving end. The main application of such two-hop wireless link lies in ad-hoc network, wireless sensor network, cooperative relay of 4G and 5G mobile communications. We can extend the work applying different combining schemes: Maximal ratio combining (MRC), selection combining (SC), and equal gain combining to enhance the performance of the communications systems. Entire result of the project work is analytical. Simulation can be done on the network considering channel gain of individual antenna element as a random variable then the concept of mixed random variable can be applied on the aggregate MIMO link. Developing such simulation, the analytical result will be verified with simulation in future.

REFERENCES

[1] Yi Zhu, Chong Tang, Lixing Song, Qingmei Yao, Shaoen Wu, "Cooperative Binary Relaying and Combining for Multi-hop Wireless Communication", Globecom 2012 - Wireless Communications Symposium, 978-1-4673-0921-9/12/$31.00 ©2012 IEEE 4421.

[2] Guangping Li, Steven D. Blostein and Jiayin Qin, "Performance Analysis of Two- Hop OSTBC Transmission over Rayleigh Fading Channels", EURASIP Journal on Wireless Communications and Networking, Volume 2010, Article ID 649541, 8 pages.

[3] Mr. Alhaj-Saleh Abdallah, Prof. Mohamed Zohdy, " Evaluation of EKF Based Receiver for Bluetooth in Presence of IEEE 802.11 Interference", International Journal of Advanced Research in Computer and Communication Engineering Vol. 5, Issue 2, February 2016.

[4] Sarwar Jahan, Md Imdadul Islam, Mohamed Ruhul Amin, Performance Evaluation of Two-Hop Wireless Network under Asymmetric Fading Environment, Received 29 October 2015; accepted 4 December 2015; published 8 December 2015 Copyright © 2015 by authors and Scientific Research Publishing Inc. , Journal of Computer and Communications, 2015, 3, 21-27 Published Online December 2015 in SciRes. http://www.scirp.org/journal/jcc http://dx.doi.org/10.4236/jcc.2015.312003.

[5] Himal A. Suraweera, *Member, IEEE*, Raymond H. Y. Louie, *Student Member, IEEE*, Yonghui Li, *Member, IEEE*,George K. Karagiannidis, *Senior Member, IEEE*, and Branka Vucetic, *Fellow, IEEE, "*Two Hop Amplify-and-Forward Transmission inMixed Rayleigh and Rician Fading Channels", IEEE COMMUNICATIONS LETTERS, VOL. 13, NO. 4, APRIL 2009.

[6] A. Sudhir Babu, Dr. K.V Sambasiva Rao, " Evaluation of BER for AWGN, Rayleigh and RicianFading Channels under Various Modulation Schemes", International Journal of Computer Applications (0975 – 8887)Volume 26– No.9, July 2011.

[7] Sumayya Ali, " Performance Evaluation of Fading Channels In Non-Regenerative Communication Protocol", International Journal of Science and Research (IJSR) ISSN (Online): 2319-7064 Index Copernicus Value (2013): 6.14 | Impact Factor (2013): 4.438.

[8] Oluwatobi Olabiyi, Annamalai Annamalai, " UNIFIED ANALYSIS OF TWO-HOP COOPERATIVEAMPLIFY-AND-FORWARD MULTI-RELAYNETWORKS", International Journal of Computer Networks & Communications (IJCNC) Vol.4, No.6, November 2012.

[9] Ahasanun Nessa, Qinghai Yang, Kyung-sup Kwak, " Performance Analysis of Two-Hop Cooperative MIMO Transmission with Best Relay Selection in Rayleigh Fading Channel", The International Arab Journal of Information Technology, Vol. 8, No. 1, January 2011

[10] Abu Sayed Md. Mostafizur Rahaman, Md. Imdadul Islam, M.R. Amin, "Selection of the Best Two-Hop AF Wireless Link under Multiple Antenna Schemes over a Fading Channel", J Inf Process Syst, Vol.11, No.1, pp.57~75, March 2015.

[11] Hang Liu, Lin Luo, Dengyuan Wu,Jiguo Yu,Dechang Chen," Routing, spectrum access, and scheduling in multi-hop multi-channel wireless networks with MIMO links" EURASIP Journal on Wireless Communications and Networking20152015:65,DOI: 10.1186/s13638-015-0252-2.

[12] D Wang, S Yang, Y Hu, D Teng, in Proceedings of International Conference on Wireless Communications, Networking and Mobile Computing. 9square: A new architecture for mimo wmn (Wuhan, China, 2006).

[13] J Wang, D Peng, W Jia, L Huang, H Li, "Joint bandwidth allocation, element assignment and scheduling for wireless mesh networks with MIMO links". Elsevier J. Comput. Commun. 31(7), 1372–1384,2008.

[14] M Cao, X Wang, S Kim, M Madihian, Multi-hop wireless backhaul networks: A cross-layer design paradigm. IEEE J. Selected Areas Commun25. 4, 738–748,2007.

[15] Oladayo Salami, Antoine Bagula, H.Anthony Chan, "Framework for link reliability in inter-working multi-hop wireless networks, Communication Research Group, Electrical Engineering Department, University of Cape Town, Rondebosch, Cape Town, South Africa, http://dx.doi.org/10.1016/j.mcm.2010.08.027.

[16] Shivali G. Bansal and Jemal H. Abawajy, "Perfomance analysis of two-hop decode-amplify-forward relayed system in different fading conditions", WIRELESS COMMUNICATION AND MOBILE COMPUTING, Vol.15:649-665

[17] Senaratne D, Tellambura C. Unified performance analysis of two-hop amplify and forward relaying, In Proceedings of IEEE Communications Society, ICC, 2009.

[18] Simon Haykin and Michael Mohs, "Modern Wireless Communications", Pearson Education, 2005.