Performance evaluation of alternate routing network based on MMPP traffic model

A Thesis submitted

by

Sadia Ahmed 2008-03-98-002
Md. Rifat hossain Khan 2009-02-98-013

Under the supervision of

Dr Mohammed Ruhul Amin
Professor,
Dept. of Electronics and Communications Engineering
East West University

Dr Md. Imdadul Islam
Associate Professor (Adjunct),
Dept. of Electronics and Communications Engineering
East West University

Department of Electronics and Communications Engineering
East West University

Fall Semester 2009 – Spring 2010
March 2010

Declaration

This is to certify that this project is our original work. No part of this has been submitted elsewhere partially or fully for the award of any other degree. Any material reproduced in this project has been properly acknowledged.

Student's names & Signatures

Name:	Sadia Ahmed
ID:	2008-03-98-002
Department:	ECE

Name:	Md. Rifat hossain Khan
ID:	2009-02-98-013
Department:	ECE

Acceptance

The paper is submitted to the department of Electronics and Communication Engineering of East West University & accepted as to style and content for the degree requirement of Master of Science in Telecommunication Engineering.

Dr Mohammed Ruhul Amin
Professor,
Dept. of Electronics and Communications Engineering
East West University

Dr Md. Imdadul Islam
Associate Professor (Adjunct),
Dept. of Electronics and Communications Engineering
East West University.

ACKOWLEDGEMETS

First of all we would like to express our gratefulness to our supervisor, Prof. Dr. Mohammad Ruhul Amin & Md. Imdadul Islam, for providing us their valuable guidance and time for this project, for introducing us to the world of teletraffic engineering and networking.

We would also like to thank our parents for supporting and encouraging us all the time.

Finally, we express our heartiest gratefulness to the almighty God. Without His divine blessing it would never possible for us to complete this project successfully.

ABSTRACT

To enhance performance of a network, in context of throughput overflow traffic between two nodes needs to be passed through one or more alternate paths. In recent literatures, alternate path is mostly analysis done based on Equivalent Random Theory (ERT), extended ERT and cost optimization technique. In this thesis, we apply the concept of Automatic Repeat Request (ARQ) Scheme of wireless communications in alternate routing traffic to get 4-state State Transition chain. Finally the state transition chain is converted to a 2-State transition chain is converted to 2-State Markov Modulated Poison Process (MMPP) to determine performance of the network in terms of MMPP traffic parameters.

Contents **Page No.**

Chapter 3

Result

Chapter 4

Conclusion

Chapter 1
Introduction

1.1 Overview

Routing is a key element of any communication network functional structure, which has a decisive impact on network traffic performance and cost. A routing method is primarily concerned with the definition of a route, or set of routes, between a pair of nodes satisfying some optimality criteria.In communication system, alternative routing or alternate path is used to transfer excess traffic. It makes overflow systems reliable and thereby reduces the possibility of congestion. Because these networks carry large amounts of traffic, alternate routing methods are designed in order to allow traffic to be properly re-routed from source to destination in the event of certain events, such as link blocking or failure.

Alternate path analysis is mostly done based on Equivalent Random Theory (ERT), extended ERT and cost optimization technique. In our thesis, the concept of alternate routing traffic is applied to get 4-state State Transition chain. After that,3 nodes of 4 states are super-posted in one node. Finally the state transition chain is converted to a 2-State transition chain is converted to 2-State Markov Modulated Poison Process (MMPP) to determine performance of the network in terms of MMPP traffic parameters.

A Markov-Modulated Poisson Process (MMPP) is a state dependent arrival process which is a doubly stochastic process where the intensity of a Poisson process is determined by the states (phases) of a Markov chain. The Markov chain can therefore be said to modulate the Poisson process which is also a stochastic process in which events/arrivals/births/calls occur continuously and independently of one-another. This modulation of MMPP introduces correlations between successive inter-arrival times in the process. The MMPP can be identified as a special case of the Markovian arrival process (MAP). The main advantage of using MMPPs as traffic models is that they helps to provide better analysis than some competing models. Important properties of queuing systems like MMPP/G/1 can be, not very easily, derived. Because of its tractability it is widely used for modeling bursty traffic such as packetized voice in the ATM.

1.2 Alternate Routing Traffic

When traffic can't be served by the initial trunk group attempted, some networks support alternative trunk groups as overflow groups. The design problem is that the traffic to the overflow group is no longer random, causing peaks of activity. There are several models of Alternate Routing traffic like as on Equivalent Random Theory (ERT),

The basis of ERT is that peaked traffic can be modeled as overflow traffic from a trunk group that has been offered random traffic. What is needed then to estimate the original offered traffic from the overflow traffic erlangs.

The problem is to determine the number of trunks required, when the traffic is peaked, i.e. a Variance To Mean Ratio (VMR) greater than 1. The VMR measures traffic peakedness. It is particularly useful for calculating skewness of nonrandom traffic (e.g., overflow route traffic) and is obtained with the ratio of variance of the offered load to average of the offered load. The solution was a model better than Erlang B, designed to solve trunk sizing when the traffic is random (VMR=1).

Consider the following figure, Random traffic is offered to a first attempt trunk group, some or most is carried, and the rest overflows to the overflow group. What is known is the overflowed amount, but what is needed to be known is the original offered load.

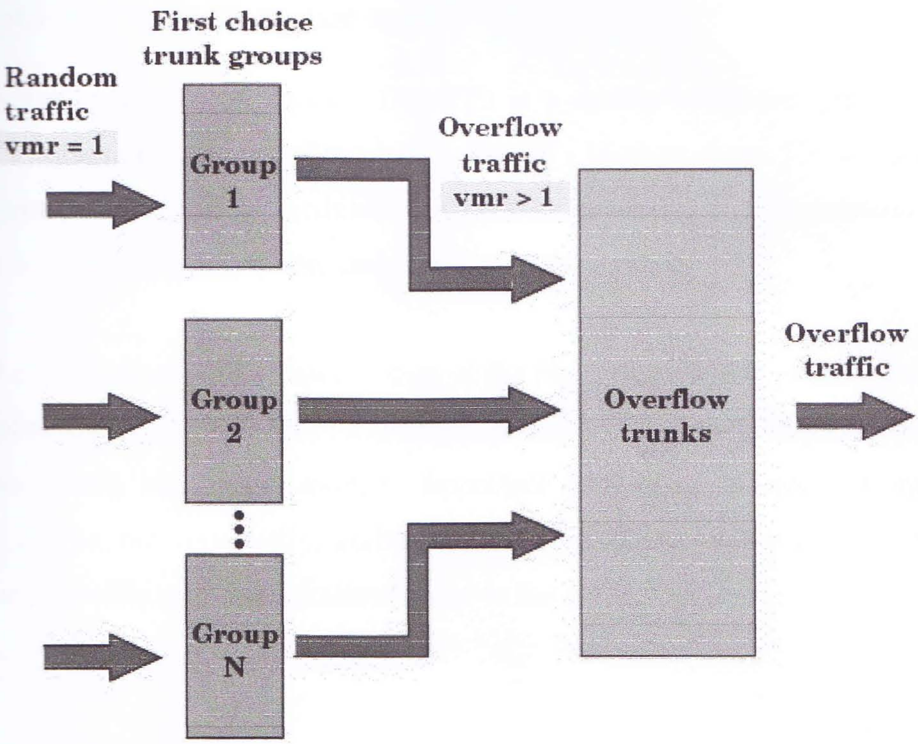


Fig.1.1 ERT Concept Flow Chart

The Equivalent Random Theory model requires the VMR of the offered traffic to be specified. If the offered traffic is overflow from a trunk group that has been sized using the Erlang B model, the following relationships apply:

$$AvgOverflow = E(A,N) \times A$$

$$Variance - of - Overflow = Avg \times \left(1 - Avg + \frac{A}{N + 1 + Avg - A}\right)$$

Where, E(A,N) = Probability of blocking from Erlang B model

A = Offered Traffic

N = Number of Trunks

Avg = Avg of Overflow Traffic

1.3 Advantage of MMPP traffic model

A Markov-modulated Poisson Process (MMPP) is a doubly stochastic process where the intensity of a Poisson process is defined by the state of a Markov chain. The Markov chain can therefore be said to modulate the Poisson process. This modulation introduces correlations between successive inter-arrival times in the process.

The MMPP can be identified as a special case of the Markovian arrival process (MAP). The main advantage of using MMPPs as traffic models is that they helps to provide better analysis than some competing models. Important properties of queuing systems like MMPP/G/1 can be, not very easily, derived. Because of its tractability it is widely used for modeling bursty traffic such as packetized voice in the ATM.

1.4. MAP classification:

Markov arrival process

One of the major drawback Markov chain lies in incorporation of large number of probability states which complicates the analysis traffic parameters of a network. Markov arrival process MAP provides and equivalent state transition chain of few probability states with some assumption [1].

Let us consider a CTMC of states $S = \{1, 2, 3, \ldots, N\}$ to be irreducible, stationary and time homogeneous. The Markov chain stays in a state $i \in S$ for an interval follows negative exponential pdf with a mean value of $1/\mu_i$ (μ_i is the termination rate of state i). After elapsing sojourn time in state i, the process makes a transition to another state $j \in S$ with probability c_{ij} for the case of without arrival or with a probability d_{ij} for the case of with arrival. In this case,

$$\sum_{\substack{j\in S \\ j\neq i}} c_{i,j} + \sum_{j\in S} d_{i,j} = 1 \text{ for } i \in S$$

Markov arrival process of order N, MAP(N) is defined by two $N \times N$ matrices **C** and **D.** Here **D** is a non negative matrix which corresponds to transition rates of states (N states) of background CTMC associated with arrival. The matrix **C** correspond to transition rates without arrival where **C** has negative diagonal elements and nonnegative off-diagonal elements. If $\boldsymbol{\pi}$ is the stationary probability vector of the underlying Markov chain then,
$\boldsymbol{\pi}$(**C**+**D**)=0 and $\boldsymbol{\pi}$.**e**=1; where **e** is a column vector whose all elements are one. The element $C_{i,j}$ of **C** matrix,

$$C_{i,j} = \begin{cases} -\mu_i; & for\ i = j \\ \mu_i c_{i,j}; & Otherwise \end{cases}$$ and the element of **D** matrix, $D_{i,j}=d_{i,j}\mu_i$.

Let us consider a continuous time Markov process with state space {1, 2, 3 ,…, m+1} where states 1, 2, 3, 4, …m are transient states and m+1 is the absorbing state. Starting from any state i, the process must enter to the absorbing state m+1. The process upon entering the absorbing state, it instantaneously jumps to a transient sate j, j=1, 2, 3 , ..., m. Let $p_{i,j}$ be the probability that the process enters the absorbing state from state i and is immediately restarted in state j, $1 \leq i, j \leq m$ and $q_{i,j}$ be the probability that the process enters another state j from i, without being absorbed satisfies,

$$\sum_{\substack{j=1 \\ j \neq i}}^{m} q_{i,j} + \sum_{j=1}^{m} p_{i,j} = 1; \ 1 \leq i \leq m$$

In this case The element $C_{i,j}$ of **C** matrix,

$$C_{i,j} = \begin{cases} -\mu_i; & for\ i = j \\ \mu_i q_{i,j}; & Otherwise \end{cases}$$ and the element of **D** matrix, $D_{i,j}=p_{i,j}\mu_i$.

Markov Arrival Process (MAP) can also be defined as a process $(N(t), J(t))$ for $t \geq 0$ on the state space $\{(i, j); i \geq 0, 1 \leq j \leq m\}$ where $N(t)$ is a counting process of "arrivals", indicated the number of arrival in $(0, t]$ and $J(t)$ is a Markov process with a finite state space, $1 \leq j(t) \leq n$ of the underlying Markov chain [2].

Probability state $J(t)$ and impact of D_{ij} and C_{ij} on MAP can be understood quite comfortably including using the line diagram of fig.1.

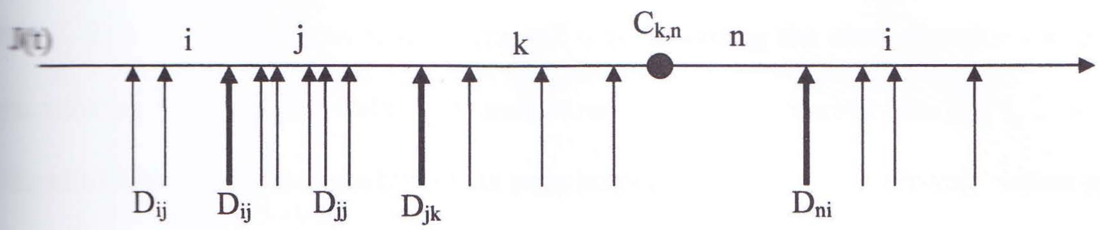


Fig.1.2 Markov Arrival Process

The MAP possesses the property of superposition i.e. superposition of n independent MAPs $\{C_i, D_i\}$; $1 \le i \le n$ is another MAP where **C** and matrix **D** matrices of resultant MAP are, $\mathbf{C}=\mathbf{C}_1 \oplus \mathbf{C}_2 \oplus \ldots \oplus \mathbf{C}_n$ and $\mathbf{D}=\mathbf{D}_1 \oplus \mathbf{D}_2 \oplus \ldots \oplus \mathbf{D}_n$; where $\oplus$ denotes the matrix Kronecker sum defined as, $A \oplus \mathbf{B}=\mathbf{A} \otimes \mathbf{I}_A + \mathbf{B} \otimes \mathbf{I}_B$; $\mathbf{I}_X$ is an identity matrix of size X. The Kronecker product is defined as,

$$\mathbf{U} \otimes \mathbf{V} = \begin{bmatrix} u_{11}\mathbf{V} & u_{12}\mathbf{V} & \cdots & u_{1m}\mathbf{V} \\ \vdots & \vdots & \ddots & \vdots \\ u_{n1}\mathbf{V} & u_{n2}\mathbf{V} & \cdots & u_{nm}\mathbf{V} \end{bmatrix}.$$

Teletraffic engineering adopts three most widely used cases of MAP are: PH Markov renewal process (PH-MRP), Markov Modulated Markov Process (MMPP) and Batch Markovian arrival process (BMAP).

1.4.1 Phase-type Renewal Process

Let us consider a continuous time Markov chain with state space {0, 1, 2, 3,..., m} where states 1, 2, 3, 4, ...m are transient states and 0 is absorbing the state. Assume the process upon entering the absorbing state, it instantaneously jumps to transient sate j, j=1, 2, 3 , ..., m with probability α_j and independent of its state immediately before the arrival; where α_j is an element of probability the vector **α.** A PH-renewal process is represented as PH(**α, T**) where **T** is the m×m transition matrix (the matrix of transition rates among the phases or states, T is non singular i.e. $\mathbf{T}^{-1}$ exits) and the row vector **α** with component α_j is called the initial probability vector [3-4]. A column vector T^0 is defined as, T^0=-Te; represents the transition rate from transient states to the absorbing state (T^0 represents the vector of transition rates from the transient sates {1, 2, 3, ... ,m} to the absorption sate 0), where e is the unit column vector with all components equal to 1.

Let the initial phase probability distribution be (α_0, **α**) = ($\alpha_0, \alpha_1, \alpha_2, ..., \alpha_m$) where $\sum_{i=0}^{m} \alpha_i = 1$ and the infinitesimal generator be **Q**. PH distributions are modeled as the time until absorption in a Markov Process with a single absorption state. The random variable *U*, defines as the time absorption (*U*=time until absorption), is said to have a continuous phase-type distribution. The infinitesimal generator for the Markov chain can be written in block-matrix form as,

$$\mathbf{Q}=\begin{bmatrix} 0 & \mathbf{0} \\ \mathbf{T}^0 & \mathbf{T} \end{bmatrix}$$

Here **0** is a 1×m vector of zeros, **T**=*[Tij]* is a $m \times m$ matrix following,

i) $T_{ij} \geq 0$ and $\sum_{j=1}^{p} T_{ij} \leq 0;\ 1 \leq i \leq m$

ii) $Tii<0$ and $T_{ii} \le -\sum_{\substack{j=1 \\ j \ne i}}^{p} T_{ij}$

iii) $\mathbf{T_0} = \mathbf{-Te}$ where e is a m×1 vector of ones.

The PH distribution is said to have a representation (**α**, **T**) of order p. The distribution function of (**α**, **T**),

$$F(u) = \begin{cases} \alpha_0, & u = 0 \\ 1 - \boldsymbol{\alpha} e^{\mathbf{T}u}\mathbf{e}, & u > 0 \end{cases}$$

Differentiation w.r.t. u the pdf,

$$f(u) = -\boldsymbol{\alpha} e^{\mathbf{T}u}\mathbf{Te}$$

PH renewal process with representation (**α** ,**T**) is a MAP with, **D=(-Te)α** and **C=T**. The phase-type renewal process contains many familiar arrival process including Erlang and hyper-exponential arrival process.

Case-1

Let $\boldsymbol{\alpha} = [1]$ and $\mathbf{T} = [-\lambda]$

$\therefore f(u) = \lambda e^{-\lambda u}$ becomes the pdf of negative exponential

Case-2

Let $\mathbf{T} = \begin{bmatrix} -\lambda_1 & 0 & \dots & 0 \\ 0 & -\lambda_2 & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & -\lambda_p \end{bmatrix}$, $\boldsymbol{\alpha} = (\alpha_1 \quad \alpha_2 \quad \dots \quad \alpha_p)$, $\alpha_i > 0$ and $\sum_{i=1}^{p} \alpha_i = 1$

The pdf becomes, $f(u) = \sum_{i=1}^{p} \alpha_i \lambda_i e^{-\lambda_i u}$ known as hyper exponential pdf.

Case-3

Let $\mathbf{T} = \begin{bmatrix} -\lambda & \lambda & 0 & \cdots & 0 \\ 0 & -\lambda & \lambda & \cdots & 0 \\ 0 & 0 & -\lambda & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -\lambda \end{bmatrix}$, $\boldsymbol{\alpha} = (1 \quad 0 \quad \dots \quad 0)$

The pdf becomes, $f(u) = \frac{\lambda^{p} u^{p-1} e^{-\lambda u}}{p!}$ known as p-phase Erlang pdf.

1.4.2 Batch Markov Arrival Process

The **Batch Markovian Arrival Process** (*BMAP*) is a special case of MAP which has arrivals of size greater than one. In BMAP several paths exists between state i and j, The transaction from any state i to another state j $\in$ S={0, 1, 2 ,..., N} depends on the size of arrival (batch size). Let us consider a continuous-time Markov chain of N+1 sates, S={0, 1, 2 ,..., N}, where the states L = {1, 2,...,N} are transient states and state 0 is the absorbing state. The system of CTMC evolves until the system falls in the absorption state, 0. After absorption the chain is then instantaneously restarted in one of the transient states L = {1, 2,...,N}. Let the BMAP (after absorption) starts from a transient state j, the probability for selecting the state j depends on the state i from which absorption has occurred. Thus the distribution of next arrival may depend on previous history [5].

Let the BMAP of M+1 different possible batch size and it is in a transient state i. When sojourn time of the state i is elapsed then there are M+1 possible cases of making a state transition. The probability $P(m)_{i,j}$ indicates that the BMAP enters the absorbing state 0 from the state *i* and instantaneously restated in state *j* with batch size *m* $(1 \leq m \leq M)$. Similarly the probability $P(0)_{i,j}$ indicates that the BMAP enters the absorbing state 0 from the state *i* and instantaneously restated in state *j*, $i \neq j$, without arrivals.

$P_{i,j}(m)$ satisfies, $\sum_{\substack{j \in S \\ j \neq i}} P_{i,j}(0) + \sum_{j \in S} \sum_{m \neq 1}^{\infty} P_{i,j}(m) = 1$ for all $j \in S$

Here $D(0)_{i,j}=\lambda i.P(0)_{i,j}$ for $i \neq j$, $D(0)_{i,i}=-\lambda i$ and $D(m)_{i,j}=\lambda i.P(m)_{i,j}$. In a BMAP, **D(0)** is the rate matrix of transition without arrival and **D(m)** is the rate matrices of transition with arrival of batch size m.

The matrix **D**(0) has negative diagonal elements and positive off-diagonal element and matrices **D**(m) have non-negative elements [6]. The summation of **D**(m) provides the infinitesimal generator matrix, $\mathbf{D} = \sum_{m=0}^{\infty} \mathbf{D}(m)$.

Let $\boldsymbol{\pi}$ be the stationary probability of the Markov process then $\boldsymbol{\pi}\mathbf{D} = 0$ and $\boldsymbol{\pi}\mathbf{e} = 1$; where **e** is a column vector of 1's and π_j is the stationary probability that the arrival process in state j.

The stationary arrival rate of the process, $\lambda = \boldsymbol{\pi} \sum_{m=1}^{M} m\mathbf{D}(m).\mathbf{e}$.

The cumulative distribution function of the inter-arrival time for the batch size m is,

$\mathbf{F(t)} = \boldsymbol{\pi}(1 - e^{\mathbf{D}(0)t})(-\mathbf{D}(0))^{-1}.\mathbf{D}(\mathrm{m}).\mathbf{e}$

1.4.3 Markov Modulated Poisson Process

Markov Modulated Markov Process (MMPP) is a doubly stochastic process whose arrival rate is given by $\lambda[J(t)] \geq 0$ where J(t), $t \geq 0$, is an m-state irreducible Markov process. The arrival rate $\lambda(t)$ at time t is modulated by the CTMC J(t) i.e. if J(t) is i, then the arrivals are according to a Poisson process with rate λ_i. The arrival rate takes on only m values λ_1, λ_2, ...,

λ_m and equal to λ_j whenever the Markov process is in the state j. If the underlying Markov process has infinitesimal generator Q of dimension m and a diagonal matrix of same dimetion, Λ=diag($\lambda_1, \lambda_2, \ldots, \lambda_m$) then we can consider Markov Modulated Markov Process is the special instance of an MAP with C = Q- Λ, D = Λ. We use (**Q**, **Λ**) to represent the MMPP. Let us consider the transition and diagonal matrix of dimension m,

$$\mathbf{Q} = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1m} \\ q_{21} & q_{22} & \cdots & q_{21m} \\ \vdots & \vdots & \ddots & \vdots \\ q_{m1} & q_{m2} & \cdots & q_{mm} \end{bmatrix} \text{ and } \mathbf{\Lambda} = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \lambda_m \end{bmatrix}$$

Probability state J(t) and impact of q_{ij} on λ_i of MMPP can be understood quite comfortably including using the line diagram of fig.2.

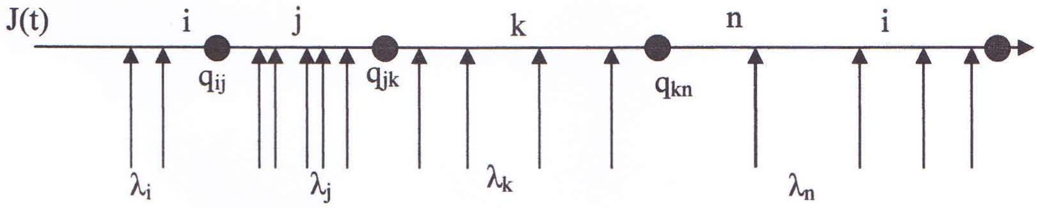


Fig.1.3 Markov Modulated Poisson Process (MMPP)

Let N_t denote the number of arrivals in [0,t] and for k≥0, $1 \le i, j \le m$, $P_{ij}(k,t)=P\{N_t= k, J_t= j|$ $N_0= 0, J_0= i\}$.

Example-1 (MMPP)

Consider a superposition of N independent on and off sources; where J(t) is the number of active sources at time t and when the state is k, arrival rate is $k\lambda$. The arrival process is, in fact, an MMPP with (Q, Λ)

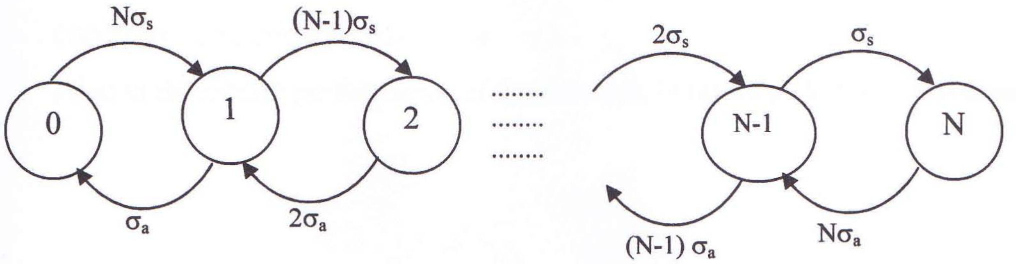


Fig.1.4 Markov Chain

$$\Lambda = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 0 & \lambda & 0 & \cdots & 0 \\ 0 & 0 & 2\lambda & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & N\lambda \end{bmatrix}$$

$$\mathbf{Q} = \begin{bmatrix} -N\sigma_s & N\sigma_s & 0 & 0 & \cdots & 0 & 0 \\ \sigma_a & -(\sigma_a + (N-1)\sigma_s) & (N-1)\sigma_s & 0 & \cdots & 0 & 0 \\ 0 & 2\sigma_a & -(2\sigma_a + (N-2)\sigma_s) & (N-1)\sigma_s & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & N\sigma_a & -N\sigma_a \end{bmatrix}$$

1.5 Objective of this Thesis

The goals were to be achieved throughout this project are as follows –

- Learn the basic of Alternate Routing for a overflow traffic exchange
- To implement the Markov Chain in Alternate Routing Traffic
- To convert the Markov Chain to the 2-State Markov Modulated Poisson Process
- To determine the under-loaded & overloaded condition of a super node with the condition state transition of nodes
- Also to determine performance of the network in terms of MMPP traffic parameters

1.6 Introduction to this Dissertation

Chapter one is the introductory chapter where project overview and benefits of Alternative Routing. The objectives of thesis are also described.

Chapter two includes a small introduction on traffic model of alternate route network. Followed by, a brief description of State Transition chain of Alternate route traffic total with the model of packetizing process.

Chapter three is the result analysis of under-loaded & overloaded condition of a super node with the condition state transition of nodes using MMPP traffic parameters.

Chapter four is discussed all about the future works based on this project with limitations & conclusion.

Chapter 2
Traffic model of alternate route network

2.1 Alternate Route through Tandem

In a multi-exchange area, adjacent nodes are connected directly by high capacity links. To enhance reliability, all the nodes of the exchange are connected through a Tandem Exchange. If direct link fails to carry the offered traffic, the Tandem Exchange is used as an alternate route to carry overflow traffic.

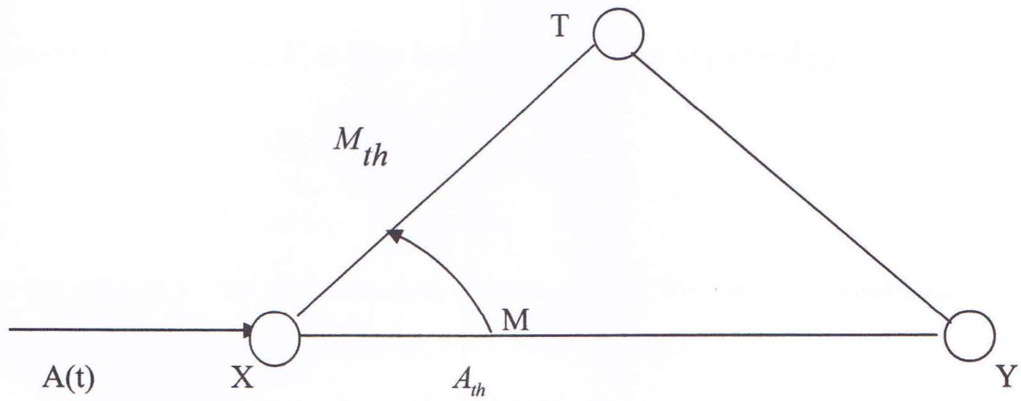


Fig.2.1 Alternate Route through Tandem Exchange

Let, Poisson's traffic $A(t)$ is offered at node X destined for the node Y. The overflow traffic M is routed through the Tandem Exchange T in the Fig.1. Here, the both X and T has some overflow traffic.

The offered traffic $A(t)$ at node X is a random variable which follows negative exponential $\left(e^{-x}\right)$ probability density function (pdf). The node X remains in under-loaded condition when $A(t) \leq A_{th}$, where A_{th} is a threshold value of the offered traffic $A(t)$, determined from long term observation of the network. From Wilkinson formula, the mean and variance of overflow traffic of link XY are

$$M = AB_n(A),$$

$$V = M\left(1 - M + \frac{A}{n+1-A+M}\right),$$

Where n is the number of trunks between node X and Y, $B_n(A)$ is the blocking probability of the link.

The overflow traffic M is carried by the link XT. The mean and variance of the overflow traffic of link XT is the lost traffic whose mean and variance are m and v respectively.

The condition of the node X, at time instant $n = k$, can be expressed as

$$X(k) = \begin{cases} G, & if \quad A \le A_{th} \\ B, & if \quad A > A_{th}. \end{cases}$$

Since the offered traffic of the tandem exchange is M, therefore, condition of the node T, at time instant $n = k$ can be expressed as,

$$T(k) = \begin{cases} G, & if \quad M \le M_{th} \\ B, & if \quad M > M_{th}, \end{cases}$$

Where M_{th} is the threshold value of the overflow traffic of link XY. M_{th} can also be determined from long term observation of the network.

2.2 State Transition chain of Alternate route traffic

Let us consider that the call arrival rate of node X during under-loaded and over-loaded conditions be λ_{Xu} and λ_{Xo} respectively. The rate of transition from under-load to over-load is r_{1X} and that of overload to underload is r_{2X}. The identical parameters are used for tandem node T but subscript is used 'T' instead of 'X'. Now the state transition diagram of node X and T are shown in Fig. 2.2 and Fig. 2.3.

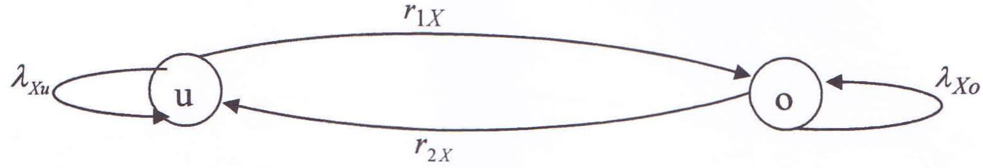


Fig.2.2 Transition from underload to overload for destined node X

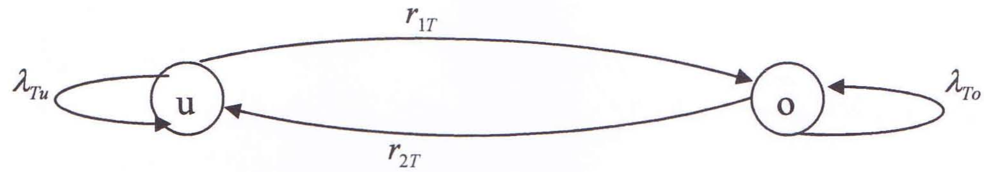


Fig.2.3 Transition from underload to overload for Tandem node Y

The combined state transition diagram of alternate path traffic model will take the form of fig.2.4

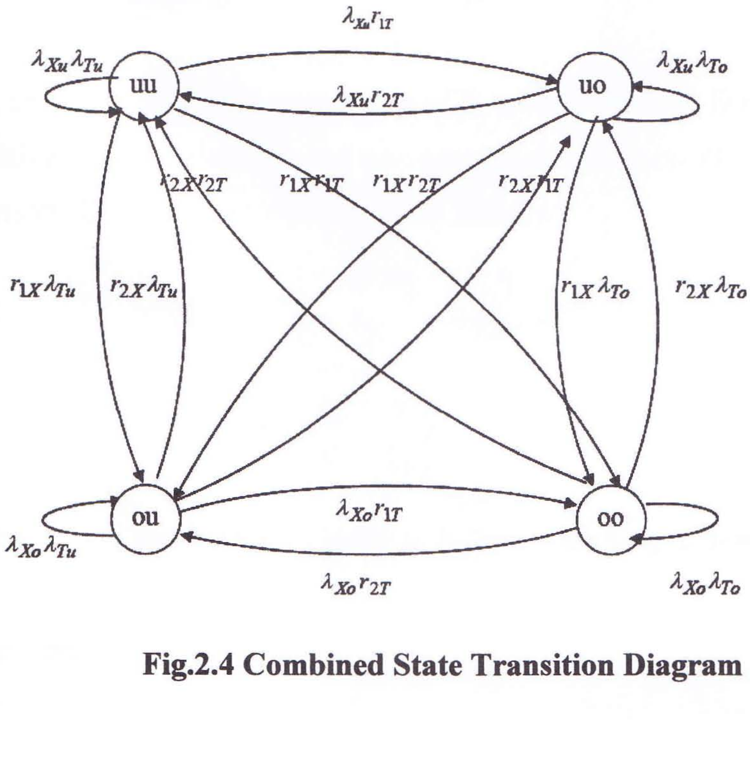


Fig.2.4 Combined State Transition Diagram

Let us convert the state transition diagram of Fig. 2.4 of 4-states to an equivalent state transition model of 2-states, like Fig. 2.5 by superposing all 3 nodes related to overload state.

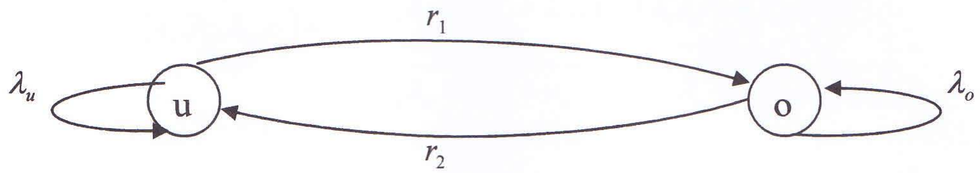


Fig.2.5 Equivalent State transition diagram

Where $\lambda_u = \lambda_{Xu}\lambda_{Tu}$

$$\lambda_o = \lambda_{Xo}\lambda_{Tu} + \lambda_{Xu}\lambda_{To} + \lambda_{Xo}\lambda_{To} + r_{1X}r_{2T} + r_{2X}r_{1T} + r_{1X}\lambda_{To} + r_{2X}\lambda_{To} + \lambda_{Xo}r_{1T} + \lambda_{Xo}r_{2T}$$

$$r_1 = \lambda_{Xu}r_{1T} + r_{1X}r_{1T} + r_{1X}\lambda_{Tu}$$

$$r_2 = \lambda_{Xu}r_{2T} + r_{2X}r_{2T} + r_{2X}\lambda_{Tu}$$

From the 2-state MMPP of Fig.2.5, Let us determine transition probability matrix,

$$T = -C^{-1}D$$

Let, here **p** be the steady state vector of T, where both C & D are M × M matrices, C has negative diagonal elements and non-negative off-diagonal elements and D has non-negative elements. C & D can be represented as follows:

$$C = \begin{bmatrix} -\lambda_u - r_1 & r_1 \\ r_2 & -\lambda_o - r_2 \end{bmatrix},$$

$$D = \begin{bmatrix} \lambda_u & 0 \\ 0 & \lambda_o \end{bmatrix},$$

where $\lambda_u > \lambda_0$ and the state transition diagram including arrivals for the MMPP (2) in the Fig. 2.5.

Therefore,

$$T(\lambda_u, r_1, \lambda_0, r_2) = -C(\lambda_u, r_1, \lambda_0, r_2)^{-1} D(\lambda_u, \lambda_0),$$

which implies

$$T(\lambda_u, r_1, \lambda_0, r_2) = -\begin{pmatrix} \frac{-(\lambda_o + r_2)\lambda_u}{(\lambda_u\lambda_o + \lambda_u r_2 + \lambda_o r_1)} & \frac{-r_1 \lambda_o}{(\lambda_u\lambda_o + \lambda_u r_2 + \lambda_o r_1)} \\ \frac{-r_2 \lambda_u}{(\lambda_u\lambda_o + \lambda_u r_2 + \lambda_o r_1)} & \frac{-(\lambda_u + r_1)\lambda_o}{(\lambda_u\lambda_o + \lambda_u r_2 + \lambda_o r_1)} \end{pmatrix}$$

Let us, determine Eigenvalues σ_1 and σ_2 of T matrix are

$$\sigma_1 = 1,$$

and

$$\sigma_2 = \frac{\lambda_o \lambda_u}{(\lambda_u\lambda_o + \lambda_u r_2 + \lambda_o r_1)}$$

Let us, determine auto-covariance function

$$C[k] = C[1]\sigma^{k-1}, \quad k = 1, 2, \cdots\cdots,$$

where

$$C[k] = \mathbf{p}C^{-2}D(T^{k-1} - ep)C^{-2}D\,\mathbf{e}$$

Modulation of MMPP introduces correlations between successive inter-arrival times. Now, the mean inter arrival time m of MMPP parameters, for k^{th} moment in generalized form,

$$m_k = E\{X^k\} = k!\,p(-C)^{-k+1}De, \quad \text{k=1, 2...}$$

The arrival rate is the number of calls that will arrive at a facility during a finite time period. The Greek letter lambda (λ) is generally used to represent arrival rate. The arrival rate under overload condition can be determined using,

$$\lambda_u = \frac{\sum_{k=w}^{0} k \cdot g[k]}{\sum_{k=w}^{0} g[k]}$$

Where $g[k] = P_r\{N = k\}$, k=0, 1, 2.......

The k^{th} moment, m_k, & auto-*covariance* function, $C[k]$ are all known statistical parameter of a system from its long term observation. From above relations $r_1, r_2, \lambda_u, \lambda_o$ can be determined.

Here, for steady state vector ***p***, we know

$$pT = p$$
$$pe = 1$$

From these to equations, the steady state vector can be determined by following

$$p = \begin{bmatrix} \frac{\lambda_u r_2}{\lambda_u r_2 + \lambda_o r_1} & \frac{\lambda_o r_1}{\lambda_u r_2 + \lambda_o r_1} \end{bmatrix}$$

For $k = 1$, the value of m_k should be as follows,

$$m_1 = \begin{bmatrix} \frac{\lambda_u r_2}{\lambda_u r_2 + \lambda_o r_1} & \frac{\lambda_o r_1}{\lambda_u r_2 + \lambda_o r_1} \end{bmatrix} \bullet \begin{bmatrix} -\lambda_u - r_1 & r_1 \\ r_2 & -\lambda_o - r_2 \end{bmatrix}^{-(1+1)} \bullet \begin{bmatrix} \lambda_u & 0 \\ 0 & \lambda_o \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow m_1 = \begin{bmatrix} \frac{\lambda_u r_2}{\lambda_u r_2 + \lambda_o r} & \frac{\lambda_o r_1}{\lambda_u r_2 + \lambda_o r} \end{bmatrix} \bullet \begin{bmatrix} -\lambda_u - r_1 & r_1 \\ r_2 & -\lambda_o - r_2 \end{bmatrix}^{-2} \bullet \begin{bmatrix} \lambda_u \\ \lambda_o \end{bmatrix}$$

$$\Rightarrow m_1 = \begin{bmatrix} \frac{\lambda_u r_2}{\lambda_u r_2 + \lambda_o r_1} & \frac{\lambda_o r_1}{\lambda_u r_2 + \lambda_o r_1} \end{bmatrix} \bullet \left[\begin{bmatrix} -\lambda_u - r_1 & r_1 \\ r_2 & -\lambda_o - r_2 \end{bmatrix}^{-1} \right]^2 \bullet \begin{bmatrix} \lambda_u \\ \lambda_o \end{bmatrix}$$

$$\Rightarrow m_1 = \begin{bmatrix} \frac{\lambda_u r_2}{\lambda_u r_2 + \lambda_o r_1} & \frac{\lambda_o r_1}{\lambda_u r_2 + \lambda_o r_1} \end{bmatrix} \bullet \left[\frac{\begin{bmatrix} \lambda_o + r_2 & r_1 \\ r_2 & \lambda_u + r_1 \end{bmatrix}}{(\lambda_u + r_1)(\lambda_o + r_2) - r_1 r_2} \right]^2 \bullet \begin{bmatrix} \lambda_u \\ \lambda_o \end{bmatrix}$$

$$\Rightarrow m_1 = \left[\frac{\lambda_u r_2}{\lambda_u r_2 + \lambda_0 r_1} \frac{\lambda_u r_1}{\lambda_0 r_2 + \lambda_0 r_1} \right] \bullet \left[\frac{\begin{bmatrix} \lambda_0 + r_2 & r_1 \\ r_2 & \lambda_u + r_1 \end{bmatrix}}{(\lambda_u \lambda_o + \lambda_o r_2 + r_1 \lambda_o + r_1 r_2 - r_1 r_2)^2} \right] \bullet \begin{bmatrix} \lambda_u \\ \lambda_o \end{bmatrix}$$

$$\Rightarrow m_1 = \left[\frac{\lambda_u r_2}{\lambda_u r_2 + \lambda_0 r_1} \frac{\lambda_u r_1}{\lambda_0 r_2 + \lambda_0 r_1} \right] \bullet \left[\frac{\begin{bmatrix} \lambda_0 + r_2 & r_1 \\ r_2 & \lambda_u + r_1 \end{bmatrix}}{(\lambda_u \lambda_o + \lambda_o r_2 + r_1 \lambda_o)^2} \right] \bullet \begin{bmatrix} \lambda_u \\ \lambda_o \end{bmatrix}$$

$$\Rightarrow m_1 = \frac{1}{(\lambda_u \lambda_o + \lambda_o r_2 + r_1 \lambda_o)^2} \left[\frac{\lambda_u r_2}{\lambda_u r_2 + \lambda_0 r_1} \quad \frac{\lambda_u r_1}{\lambda_0 r_2 + \lambda_0 r_1} \right] \bullet \begin{bmatrix} \lambda_0 + r_2 & r_1 \\ r_2 & \lambda_u + r_1 \end{bmatrix}^2 \bullet \begin{bmatrix} \lambda_u \\ \lambda_o \end{bmatrix}$$

$$\Rightarrow m_1 = \frac{1}{(\lambda_u \lambda_o + \lambda_o r_2 + r_1 \lambda_o)^2} \left[\frac{\lambda_u r_2}{\lambda_u r_2 + \lambda_0 r_1} \quad \frac{\lambda_u r_1}{\lambda_0 r_2 + \lambda_0 r_1} \right] \bullet$$

$$\begin{bmatrix} (\lambda_0 + r_2)^2 + r_1 r_2 & \lambda_0 r_1 + \lambda_u r_2 + r_1 r_2 + r_1^2 \\ \lambda_0 r_2 + \lambda_u r_2 + r_1 r_2 + r_2^2 & (\lambda_u + r_1)^2 + r_1 r_2 \end{bmatrix} \bullet \begin{bmatrix} \lambda_u \\ \lambda_o \end{bmatrix}$$

$$\Rightarrow m_1 = \frac{1}{(\lambda_u \lambda_o + \lambda_o r_2 + r_1 \lambda_o)^2} \left[\frac{\lambda_u r_2}{\lambda_u r_2 + \lambda_0 r_1} \quad \frac{\lambda_u r_1}{\lambda_0 r_2 + \lambda_0 r_1} \right] \bullet$$

$$\begin{bmatrix} \{(\lambda_0 + r_2)^2 + r_1 r_2\}\lambda_u + \{\lambda_0 r_1 + \lambda_u r_2 + r_1 r_2 + r_1^2\}\lambda_o \\ \{\lambda_0 r_2 + \lambda_u r_2 + r_1 r_2 + r_2^2\}\lambda_u + \{(\lambda_u + r_1)^2 + r_1 r_2\}\lambda_o \end{bmatrix}$$

$$\Rightarrow m_1 = \frac{1}{(\lambda_u \lambda_o + \lambda_o r_2 + r_1 \lambda_o)^2} \bullet \begin{bmatrix} \frac{\lambda_u r_2}{\lambda_u r_2 + \lambda_0 r_1} \left[\{(\lambda_0 + r_2)^2 + r_1 r_2\}\lambda_u + \{\lambda_0 r_1 + \lambda_u r_2 + r_1 r_2 + r_1^2\}\lambda_o \right] + \\ \frac{\lambda_u r_1}{\lambda_0 r_2 + \lambda_0 r_1} \left[\{\lambda_0 r_2 + \lambda_u r_2 + r_1 r_2 + r_2^2\}\lambda_u + \{(\lambda_u + r_1)^2 + r_1 r_2\}\lambda_o \right] \end{bmatrix}$$

$$\Rightarrow m_1 = \frac{1}{(\lambda_u \lambda_o + \lambda_o r_2 + r_1 \lambda_o)^2} \bullet \left[\frac{(\lambda_u r_2 + \lambda_u \lambda_o + \lambda_0 r_1)^2 (r_1 + r_2)}{\lambda_u r_2 + \lambda_0 r_1} \right]$$

So the 1st inter arrival time from the above equation is

$$\Rightarrow m_1 = \frac{(r_1 + r_2)}{\lambda_u r_2 + \lambda_0 r_1}$$

2.3 MMPP traffic parameter using Modeling of Packetized Process

As we know the multi-media inputs like voice, data, image, etc are integrated in the form of packets in ISDN. And the packet is in general of variable length, in the standard packet switching system. On the other hand, it becomes a fixed length cell in the ATM for broadband ISDN. In such systems, real-time voice or video is delay-sensitive; whereas data is lo-sensitive and their performance evaluation is an important problem. Since the packetized process has a bursty nature, the superposed process from multiple sources becomes non-renewal and analyses using the MMPP have been widely applied.

A model for packetized voice is shown in figure 2.6 which is often referred to as the ON-OFF model. In a single voice source, it is assumed that voice spurt and silence periods are exponentially distributed with mean α^{-1} and β^{-1}, respectively, and packets (cells) are originated in a fixed period T during the voice spurt.

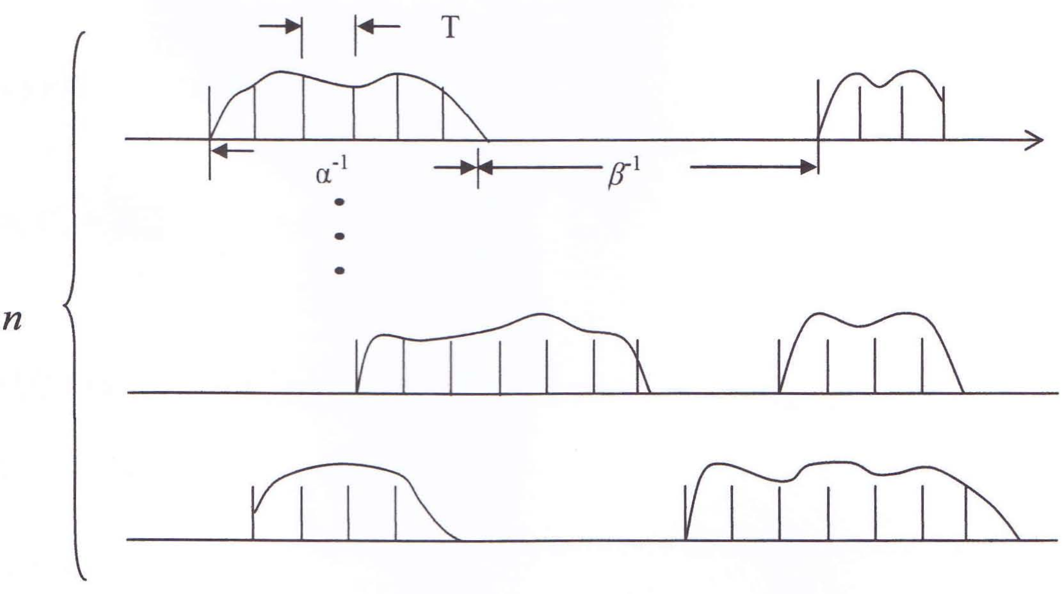


Figure: 2.6 ON-OFF model for packetized traffic

For the arrival process of the single source, the arrival rate λ_a, SCV (varianc/mean2) C_a^2 and skewness (third central moment/variance 3/2) S_k are given by

$$\lambda_a = \frac{\beta}{T(\alpha+\beta)}$$

$$C_a^2 = \frac{1-(1-\alpha T)^2}{T^2(\alpha+\beta)^2}$$

$$S_k = \frac{2\alpha T\left(\alpha^2 T^2 - 3\alpha T + 3\right)}{\alpha T(2-\alpha T)^{\frac{3}{2}}}$$

We can determine the parameters in terms of λ_a, C_a^2 and S_k

$$C \cong \left(S_k^2 + 18\right)C_a^6 - 12 S_k C_a^5 - 18 C_a^4 + 8 S_k C_a^3 + 6 C_a^2 - 2$$

The arrival rate for single source, $\lambda_n = n\lambda_a$

Let,

$$T = 16,\ \alpha = \frac{1}{352},\ \beta = \frac{1}{650}\ \&\ n = 120$$

$$S_k = 9.838\ \ C_a^2 = 18.0950$$

Now, $C_a = \sqrt{C_a^2}\quad \therefore C_a = 4.254$

So,

$$C \cong \left(S_k^2 + 18\right)C_a^6 - 12 S_k C_a^5 - 18 C_a^4 + 8 S_k C_a^3 + 6 C_a^2 - 2$$

$$\therefore C_a = 5.159 \times 10^5$$

$$C_{root} = \sqrt{C}$$

$$\therefore C_{root} = 718.28$$

$$\lambda_{H1} = \frac{\left[\lambda_a \times \left(2 - S_k C_a^3 + C_{root}\right)\right]}{\left(3C_a^4 - 2S_k C_a^3 + 1\right)}$$

$$\therefore \lambda_{H1} = 1.528 \times 10^{-3}$$

$$k_H = \frac{\left[\left(C_a^2 - 1\right)\lambda_{H1}^2\right]}{\left[\left(C_a^2 + 1\right)\lambda_{H1}^2 - 2\lambda_a\left(2\lambda_{H1} - \lambda_a\right)\right]}$$

$$\therefore k_H = 0.046$$

$$\lambda_{H2} = \frac{\left[\lambda_a \lambda_{H1}\left(k_H - 1\right)\right]}{\left[k_H \lambda_a - \lambda_{H1}\right]}$$

$$\therefore \lambda_{H2} = 0.061$$

$$\therefore k_H \cdot \lambda_{H1} = 6.978 \times 10^{-5}$$

$$a_0 = k_H \cdot \lambda_{H1} + \left(1 - k_H\right)\lambda_{H2}$$

$$\left(1 - k_H\right)\lambda_{H2} = 0.058$$

$$\therefore a_0 = 0.058$$

$$I_{\_prime0} = a_0 - \lambda_a$$

$$\lambda_a = 0.022$$

$$\Rightarrow I_{\_prime0} = 0.036$$

$$I_{\_\inf inite} = C_a^2$$

$$\Rightarrow I_{\_\inf inite} = 18.095$$

$$S_{\_\inf inite} = 3 \cdot C_a^4 - S_k C_a^3$$

$$\Rightarrow S_{\_\inf inite} = 225.035$$

$$D = \frac{I_{\_prime0}}{I_{\_\inf inite}^{-1}}$$

$$\Rightarrow D = 2.12 \times 10^{-3}$$

$$F = \frac{\left[D \cdot \left(S_{\_\inf inite}{}^{-3} \cdot I_{\_\inf inite}{}^{2}\right)\right]}{3 \cdot I_{\_\inf inite}{}^{-1}}$$

$$\Rightarrow F = 7.14 \times 10^{-3}$$

$$E = \frac{I_{\_prime0}}{F^2}$$
$$\Rightarrow E = 710.799$$

$$\lambda_n = 2.635$$

$$a = \lambda_n \cdot E$$
$$\Rightarrow a = 1.873 \times 10^3$$

$$r_1 = D \cdot \left[1 + \left(\frac{1}{\sqrt{1+a}}\right)\right]$$

$$\Rightarrow r_1 = 2.169 \times 10^{-3}$$

$$r_2 = D \cdot \left[1 - \left(\frac{1}{\sqrt{1+a}}\right)\right]$$

$$\Rightarrow r_2 = 2.071 \times 10^{-3}$$
$$\lambda_u = \lambda_n + F + \left(F \cdot \sqrt{1+a}\right)$$
$$\Rightarrow \lambda_u = 2.951$$

$$\lambda_o = \lambda_n + F - \left(F \cdot \sqrt{1+a}\right)$$
$$\Rightarrow \lambda_o = 2.333$$

Chapter 3
Result

3.1 Result of interarrival time distribution function

Analysis of under-loaded and over-loaded condition of Super node

$t = 0, 0.2, \ldots 5$

$r_1 = 2.169 \times 10^{-3}$

$\lambda_u = 2.951$

$r_2 = 2.071 \times 10^{-3}$

$\lambda_o = 2.333$

$$D = \begin{pmatrix} \lambda_u & 0 \\ 0 & \lambda_o \end{pmatrix}$$

$$C = \begin{pmatrix} -\lambda_u - r_1 & r_1 \\ r_2 & -\lambda_o - r_2 \end{pmatrix}$$

$$eigenvals(C) = \begin{pmatrix} -2.953 \\ -2.335 \end{pmatrix}$$

$$eigenvecs(C) = \begin{pmatrix} 1 & 3.509 \times 10^{-3} \\ -3.351 \times 10^{-3} & 1 \end{pmatrix}$$

$u_1 = 1.228$

$u_2 = 2.272$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

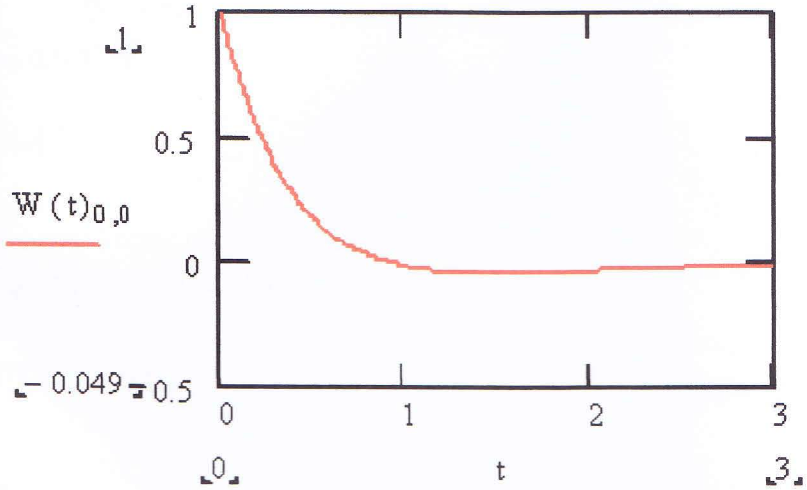
$$\alpha_0(t) = \frac{u_2 \cdot e^{-u_1 t} - u_1 \cdot e^{-u_2 t}}{u_2 - u_1}$$

$$\alpha_1(t) = \frac{e^{-u_1 t} - e^{-u_2 t}}{u_2 - u_1}$$

$$W(t) = \alpha_0(t) \cdot I + \alpha_1(t) \cdot C$$

Now, the probability matrix,

$$F(t) = (I - W(t)) \cdot (-C)^{-1} \cdot D$$



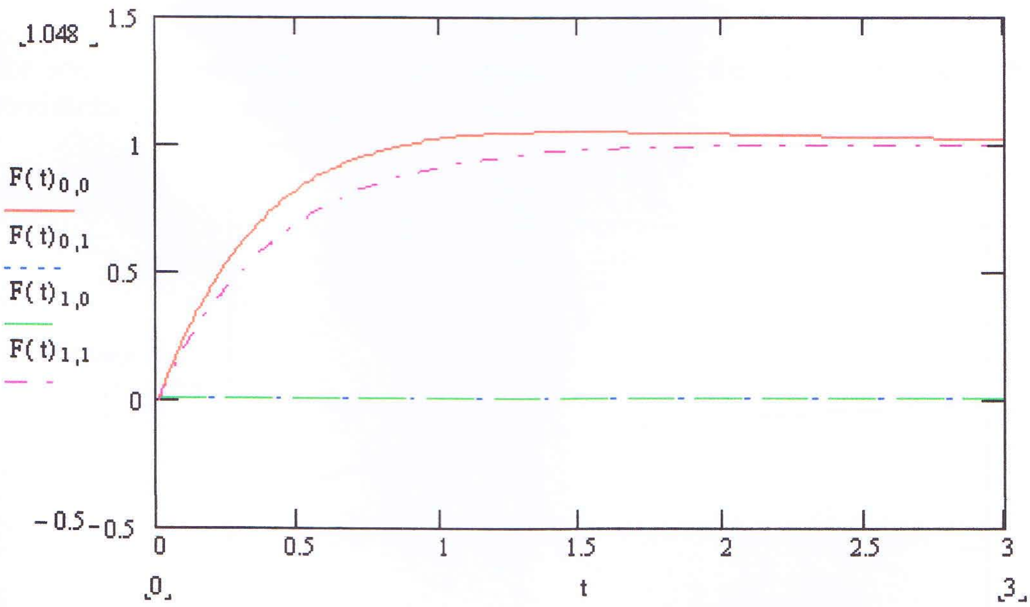


Figure3.1 Interarrival time distribution functions

3.2 Analysis of time dependant state probabilities for finite-state systems: The classical eigensystem analysis

For the finite Markov Chain, the infinitesimal generator matrix,

$$Q = \begin{pmatrix} -r_1 & r_1 \\ r_2 & -r_2 \end{pmatrix}$$

Now, eigen vector of matrix Q, $M = eigenvecs(Q)$

And eigen value of matrix Q, $Y = eigenvals(Q)$

$t = 0, 0.2, ..3$

$$Dia(t) = \begin{pmatrix} e^{Y_0 - t} & 0 \\ 0 & e^{Y_1 - t} \end{pmatrix}$$

The row vector of state probabilities,
$P(t) = (0 \quad 1) \cdot M \cdot Dia(t) \cdot M^{-1}$

$$S(t) = \sum_{i=0}^{1} P(t)_{0,i}$$

$$R(t) = \frac{P(t)}{S(t)}$$

For some discrete values of time, we get the graph for the under-loaded & over-loaded conditions of the super node.

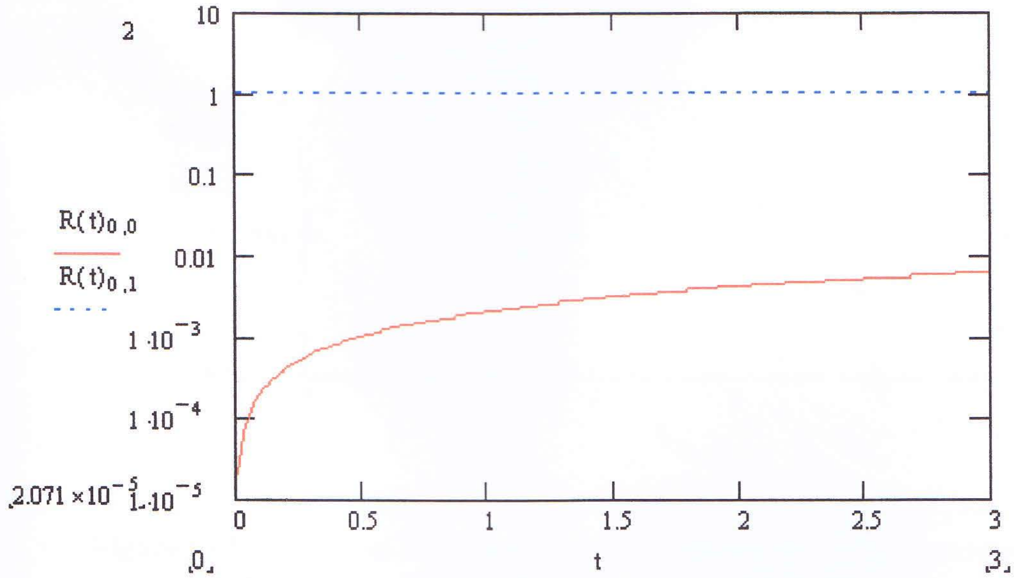


Figure3.2 Time dependant state probabilities

3.3 Determining the results with different parameter values

By considering different sampling period, ON-OFF parameters and same number of voice circuits we have assumed the MMPP traffic parameters below:

$\lambda_1=1.2 \qquad \lambda_2=1.3 \qquad r_1=0.3 \qquad r_2=0.7$

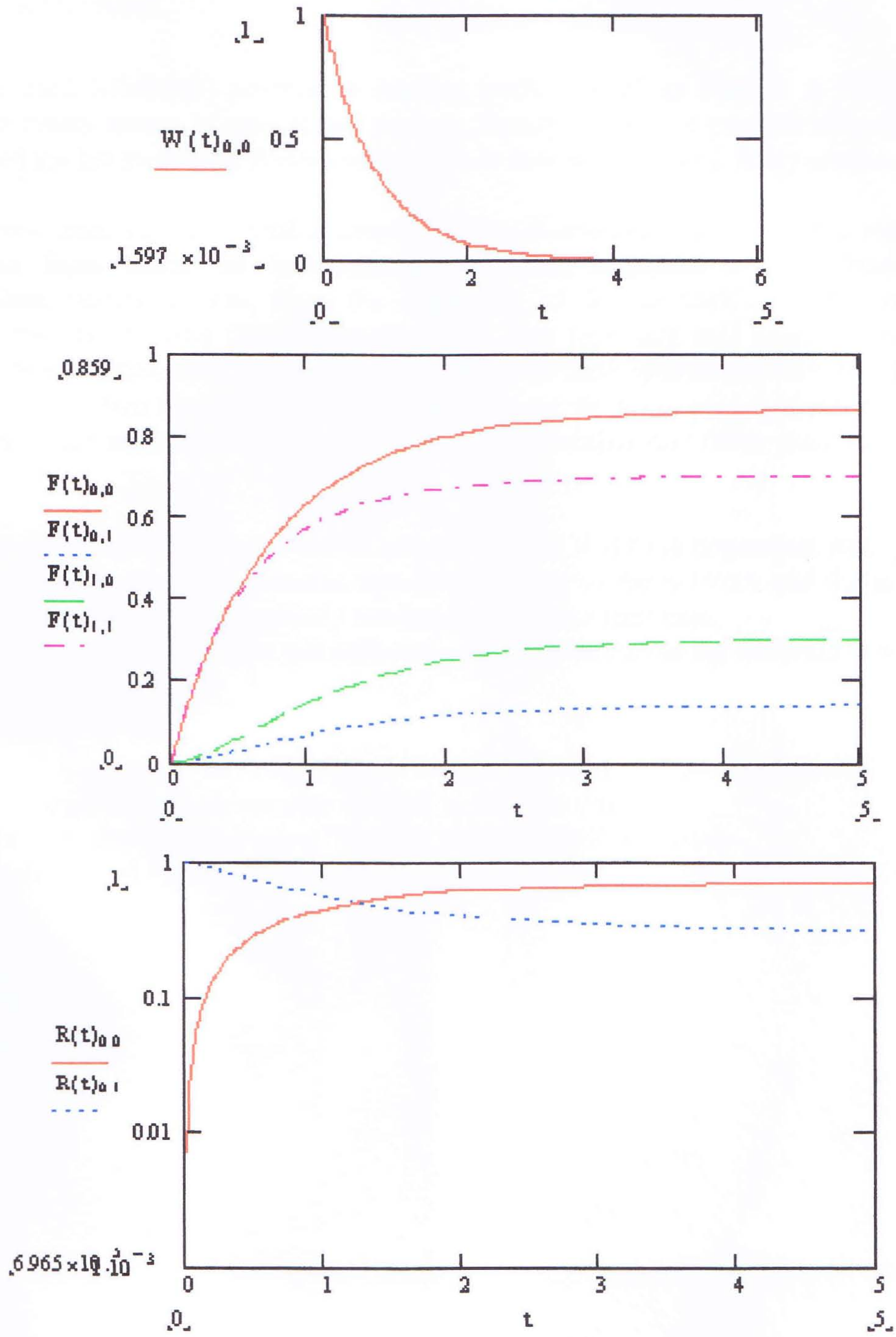


Figure3.3 Interarrival time distribution functions and time dependant state probabilities for different set of parameter values

Chapter 4
Conclusion

4.1 Conclusion:

We have used MMPP(2) alternative routing traffic model as MMPP is widely used for analyzing bursty nature of packetized process. Bursty traffic has packets of variable lengths in standard packet switching system where data is loss sensitive and delay sensitive.

We can say from figure3.1 that interarrival time distribution functions that represent state transitions from underload to overload and from overload to underload has same probabilities, which is zero, from the beginning of the network's functionality. On the contrary interarrival time distribution functions that represent self transition have slightly different probabilities at the beginning of time but as time approaches forward the functions converge. These two functions also represent different slopes of probability increments at the beginning where underload-underload reaches to probability one faster than that of overload-overload.

Moreover from figure3.2 we've drawn our conclusion that time dependant state probabilities for underload-overload state remains one from starting of the network and the same function for underload-underload state slowly reaches to 1 as time increases.
Similarly from figure3.3 we've got different performance curves for different voice circuit.

4.2 Future Work

Our future target is to apply another method to analyze time
dependant state probabilities using "RANDOMISATION" approach
Which is also called "Jensen's Method"

References

[1] Doo, IL, Choi, 'MAP/G/1/K queue with multiple threshold on buffer,' Comm. Korean math, Soc.14, no.3, pp.611-625, 1999

[2] Howon Lee and Dong-Ho Cho, 'VoIP Capacity Analysis in Cognitive radio Syatem, ' IEEE Communications Letters, vol.13, no.6, pp.393-395, June, 2009

[3] Haru Akimaru and Konosuke Kawashima, 'Teletraffic Theory and Applications,' Chapter-6, Springer-Verlag publishing, 1993

[4] Mark William Fackrell, 'Characterization of matrix-exponential distributions,' Thesis Doctor of Philosophy, University of Adelaide, School of Applied Mathematics, pp.8-26, November 3003

[5] Hiroyuki Masuyama, 'Studies on algorithmic analysis of queue with batch Markovian arrival streams,' Thesis of PhD, Applies Mathematics and Physics, Kyoto University, Japan, 2003

[6] Sadrac K. Matendo, 'Some Performance Measures for Vacation Models With a Batch Markovian Arrival Process,' Journal of Applied Mathematics and Stochastic Analysis 7, no. 2, pp.111-124, Summer 1994

[7] Sang H. Kang, Yong Han Kim, Dan K. Sung, Bong D. Choi, 'An Application of Markovian Arrival Process (MAP) to Modelling Superposed ATM cell streams,' IEEE Transactions on Communications, vol. 50, no. 4, April 2002

Abbreviations

ARQ	Automatic Repeat Request
ATM	Asynchronous Transfer Mode
BISDN	Broadband Integrated Services for Digital Network
BMAP	Batch Markovian Arrival Process
CTMC	Continuous Time Markov Chain
ERT	Equivalent Random Theory
ISDN	Integrated Services for Digital Network
MMPP	Markov Modulated Poisson Process
MMPP/G/1	Markov Modulated Poisson Process/ General/ 1
MAP	Markovian Arrival Process
PH-MRP	Phase Type- Markovian Renewal Process
VMR	Variance to Mean Ratio

List of Figures   Page