Thesis Report

“Performance Analysis of BPSK and QPSK Under Rayleigh Fading Channel”

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DECLARATION

We hereby declare that we carried out the work reported in this thesis in the Department of Electronics and Communications Engineering, East West University, under the supervision of Sarwar Jahan .We solemnly declare that to the best of our knowledge, no part of this report has been submitted elsewhere for award of any degree. All sources of knowledge used in this report have been duly acknowledged.

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CERTIFICATE

This is to certify that the thesis entitled **“**Performance Analysis of BPSK and QPSK Under Rayleih Fading Channel**”,** being submitted by Md.Momen Hasan Sudip Sarker Ashraful Alam Department of Electronics and Communications Engineering, East West University, Dhaka in partial fulfillment for the award of the degree of Bachelor of Science in Electronics and Telecommunication Engineering, is a record of major thesis carried out by them. They have worked under my supervision and guidance and have fulfilled the requirements which, to my knowledge, have reached the requisite standard for submission of this dissertation.

....

Sarwar Jahan

Assistant Professor

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Abstract

A well-defined characterization of the distortions in the wireless signal after affected by noise is analyzed to improve channel estimation and increase detection robustness. The Bit Error Rate (BER) and Signal to Noise Ratio (SNR) of the wireless signal under Rayleigh fading channel has been examined. Simulations confirmed the comparison of BER curves for Binary Phase Shift Keying (BPSK) and Quadrature Phase Shift keying (QPSK) from Adaptive White Gaussian Noise (AWGN) channel model

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Chapter 1

Introduction

Mobile communications and wireless network have experienced massive growth and commercial success in the recent years. However, the radio channels in mobile radio systems are usually not modest as the wired one. Unlike wired channels that are stationary and predictable, wireless channels are extremely random and time-variant. It is well known that the wireless multi-path channel causes an arbitrary time dispersion, attenuation, and phase shift, known as fading, in the received signal. Fading is caused by interference between two or more versions of the transmitted signal which arrive at the receiver at slightly different times [1-2].

In wireless telecommunications, multipath is the propagation phenomenon that results in radio signals reaching the receiving antenna by two or more paths. Causes of multipath include atmospheric path, ionospheric reflection and refraction, and reflection from water bodies and terrestrial objects such as mountains and buildings. The effects of multipath include constructive and destructive interference, and phase shifting of the signal. In digital radio communications (such as GSM) multipath can cause errors and affect the quality of communications [2-3].

The wireless environment is highly unstable and fading is due to multipath propagation. Multipath propagation leads to rapid fluctuations of the phase and amplitude of the signal. The presence of reflectors in the environment surrounding a transmitter and receiver create multiple paths that a transmitted signal can traverse. As a result, the receiver sees the superposition of multiple copies of the transmitted signal, each traversing a different path. Each signal copy will experience differences in attenuation, delay and phase shift while traveling from the source to the receiver. This can result in either constructive or destructive interference, amplifying or attenuating the signal power seen at the receiver. Fading may be large scale fading or small scale fading. Based on multipath time delay spread small scale fading is classified as flat fading and frequency selective fading. If bandwidth of the signal is smaller than bandwidth of the channel and delay spread is smaller than relative symbol period then flat fading occurs whereas if bandwidth of the signal is greater than bandwidth of the channel and delay spread is greater than relative symbol period then frequency selective fading occurs. Based on Doppler spread small scale fading may be fast fading or slow fading. Slow fading occurs when the coherence time of the channel is larger relative to the delay constraint of the channel. The amplitude and phase change imposed by the channel can be considered roughly constant over the period of use. Slow fading can be caused by events such as shadowing, where a large obstruction such as a hill or large building comes in the main signal path between the transmitter and the receiver. Fast fading occurs when the coherence time of the channel is small relative to the delay constraint of the channel. The amplitude and phase change imposed by the channel varies considerably over the period of use. In a fast-fading channel, the transmitter may take advantage of the variations in the channel conditions using time diversity to help increase robustness of the communication [3-5].

In a wireless mobile communication system, a signal can travel from transmitter to receiver over multiple reflective paths; this phenomenon is referred to as multipath propagation. If radio signal transmitted over the free space or multipath propagation channel contains objects (particles) which randomly scatter the energy of the transmitted signal and the scatters signal arrives at the receiver out of steps. Scatters introduces a variety of impairments including fading, shadowing, multipath delay spread, Doppler spread, attenuation etc. and the inherent background noise. These background noises can be called as thermal noise and treated as additive white Gaussian noise (AWGN). The end to end modeling and design of systems that mitigate the effects of fading are usually more challenging performance only by AWGN [3-5].

In addition to when multipath propagation occurs, radio signal scattered by objects like tress, building etc. before it arrives at the receiver then that’s time we have to use Rayleigh fading distribution channel, it’s mostly applicable radio signal distribution channel and there is no dominant line of sight radio propagation between the transmitter and the receiver [3-5].

Further added in free space or line of sight communication radio signal defiantly will loss attenuation, amplitude and phase shift changes, meantime dominant wave (time in a seconds between largest wave) can be sum of two or more, this combined signal process totally predictable process, to transmit radio signal in the long distance, dominant wave can also be subject to shadow attenuation and the mobile antenna receives a large number of reflected and scattered wave. This process or radio signal distributed by Rician fading distribution channel. This is also used in the satellite channels [3-5].

Signal to noise ratio (SNR) is a measure used in science and engineering that compares the level of a desire signal to the level of background noise. It is defined as the ratio of signal power to the noise power, often expressed in decibels. A ratio higher than 1:1 (greater than 0 dB) indicates more signal than the noise. It is clear that when the signal is higher than the interference plus the noise, the probability of decoding signal successfully is higher. One measure of performance is BER, or bit-error rate. Simply put, this is measuring how many of bits made it correctly over the wireless channel. Certain modulation schemes can deliver excellent BER performance compared to other [3-5].

Chapter 2

FADING

The communication between the base station and the mobile station in the mobile systems are mostly in non-line of sight (LOS).The LOS between the receiver and the transmitter affected by terrain and other immovable obstructed by building and others natural objects. At the mean time mobile station is also moving in different direction at different speed. The RF or wireless signal from the transmitter is scattered by reflection and diffraction and reaches the receiver through many non-LOS paths. This non-LOS path causes long-term and short term fluctuations in the form of log-normal fading, which degrades the performance of the RF or wireless channel [4-5].

2.1 Types of fading:

There are mainly 3 types of fading according to signal attenuation loss. These are –

1) Long scale fading
2) Medium scale fading
3) Short scale fading

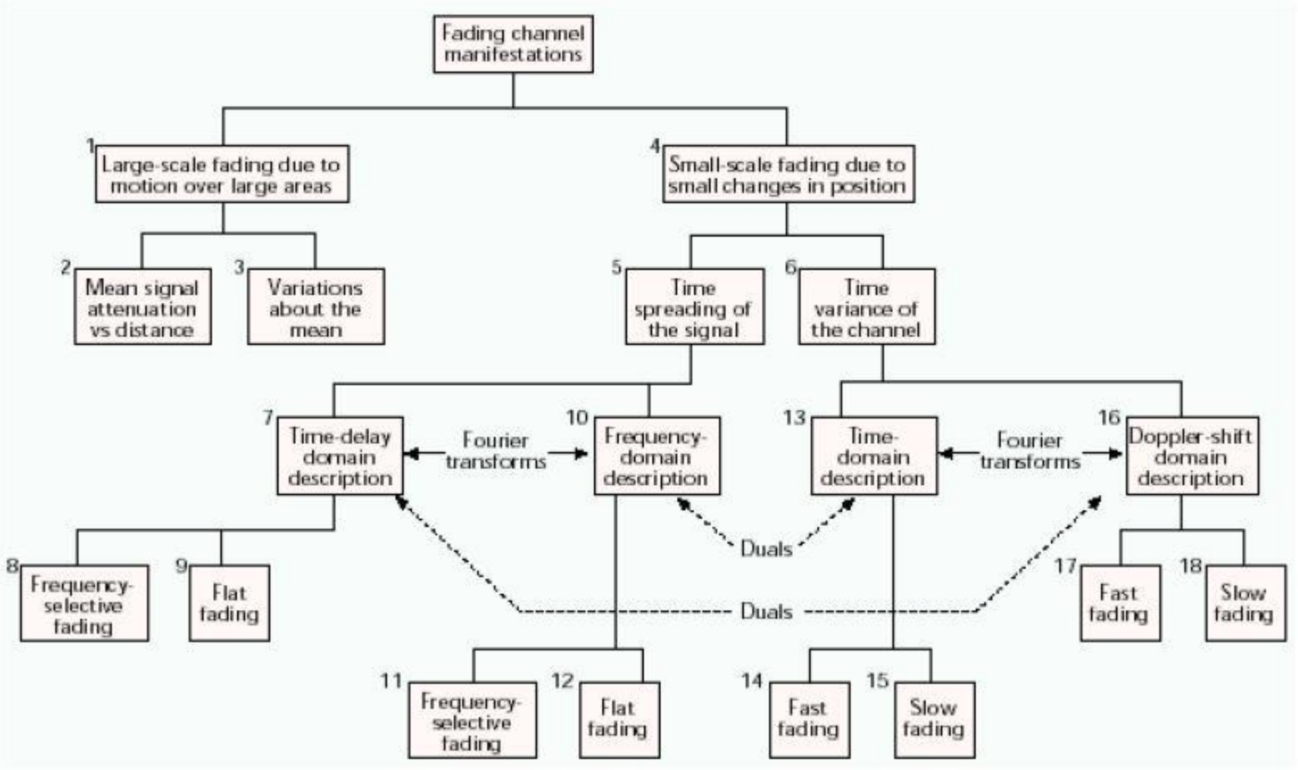

Fig 2.1: Multipath fading channel

2.2 LARGE SCALE FADING

Fig 2.1 represents the overview of fading channel in wireless mobile telecommunications. According to radio signal propagation there is two types of fading; large scale fading and small scale fading. Large scale fading represents average attenuation, amplitude and phase shift changes or path loss due to motion of the large areas. In fig.2.1 large scale fading expression is shown in blocks 1, 2 and 3. This phenomenon affected by prominent earth objects (tress, building, hills, forests etc.) between the transmitter and the receiver, the receiver is often represented as being shadowed by such prominent. Due to which a great variation occur in the strength of the received signal. The measured signal power differs substantially at different locations even though at the same radial distance from a transmitter. These Represents the medium scale fluctuations of the radio signal strength over distances from tens to hundreds of meters. Every Electromagnetic signal has its means. Average powers now when the phenomena of large scale fading occur then this value deviates from the mean average value [4-6].

2.3 Medium scale fading

Shadowing is a 'medium-scale' effect: field strength variations occur if the antenna is displaced over distances larger than a few tens or hundreds of meters.

Shadowing is the effect that the received signal power fluctuates due to objects obstructing the propagation path between transmitter and receiver. These fluctuations are experienced on local-mean powers, that is, short-term averages to remove fluctuations due to Multipath Fading.

In an area where signal reception would normally be strong, certain other factors may have an effect on the reception, thereby making it either stronger or weaker, or may cause complete RF interference. Like a building with thick walls or of mostly metal construction (or with dense rebar in concrete) may prevent a mobile phone from being used. Underground areas, such as tunnels and subway stations, lack reception unless they are wired for cell signals. There may also be gaps where the service contours of the individual base stations of one's mobile carrier (and/or its roaming partners) do not completely overlap [4-6].

2.4 Small scale fading

Small-scale fading refers to the dramatic changes in signal amplitude and phase that can be experienced as a result of small changes in the spatial separation between a receiver and transmitter. As indicated in Fig. 2.1, blocks 4, 5, and 6, small-scale fading manifests itself in two mechanisms, namely, time spreading of the signal (or signal dispersion) and time-variant behavior of the channel. For mobile radio applications, the channel is time-variant because motion between the transmitter and receiver results in propagation path changes. The rate of change of these propagation conditions accounts for the fading rapidity (rate of change of the fading impairments). Small-scale fading is also called Rayleigh fading because if the multiple reflective paths are large in number and there is no line-of-sight signal component, the envelope of the received signal is statistically described by a Rayleigh PDF. When there is a dominant non fading signal component present, such as a line-of-sight propagation path, the small scale fading envelope is described by a Rician PDF. A mobile radio roaming over a large area must process signals that experience both types of fading: small-scale fading superimposed on large-scale fading [4-6].

2.5 Flat fading

If a channel has a constant response for a bandwidth greater than the transmitted signal bandwidth, then the channel is said to be a flat fading channel.

The conditions for a flat fading channel are:

$$B_s << B_c$$

$$T_s >> T_c$$

If the baseband signal frequency is much greater than the Doppler spread then the effects of Doppler spread are negligible where Bs and Ts are the signal bandwidth and the symbol duration respectively.

The term 'Flat' indicates that all frequencies of the transmitted signal experience the same level of channel gain (or attenuation). The transmitted spectrum is preserved at receiver. The channel gain is fluctuating with time, and so is the received signal strength. Because the symbol period (*Tb*) is reciprocal of signal bandwidth, it follows that it must be much larger than the rms delay spread of the channel, which is inversely proportional to the channel coherence bandwidth (*Bc*).This type of fading channel is sometimes referred to as a 'narrow-band' channel [6-7].

2.6 Frequency selective fading

On the other hand, if the bandwidth of the transmitted signal is much larger than the channel coherence bandwidth, frequency selective fading occurs. In this case, the channel gain is not only fluctuating in time, but is also different for different frequency components of the transmitted signal. This type of fading channel is more severe than flat fading channel, as it induces Inter Symbol Interference (ISI).

$$B_s < B_c$$

$$T_s > T_c$$

The concept of pulse-shaping is used to control the transmit signal bandwidth [6].

2.7 Fast fading

Depending on how rapidly the transmitted signal changes compared to the rate of change of the channel, a channel may be classified either as fast fading or slow fading. In the fast fading channel, the channel characteristics are changing rapidly within the symbol duration. It means that the channel coherence time (Tc) is smaller than the symbol period (Tb). As the channel coherence time is inversely proportional to the Doppler spread (B_d), the fast fading channel often has high Doppler spread. In practice, fast fading only occurs for very low data rate transmission [6-7].

2.8 Slow fading

In contrast, if the channel impulse response changes at a rate much slower than the symbol rate, the channel is said to be 'slow fading'. Such channel is assumed to be static over one or several symbol periods (Tb). In frequency domain, this implies that the Doppler spread (BD) of the channel is much less than the bandwidth of the transmitted baseband signal [6-7].

2.9 Doppler spread

Doppler spread which means it describes the time dispersive nature of the channel in a local area network. Doppler spread is a measure of the spectral broadening caused by the time rate change of the mobile channel, also defined as the range of the frequency over which the received Doppler spread is essentially non zero.

When a pure sinusoidal tone of frequency f_c is transmitted, the received signal spectrum, called the Doppler spectrum, will have components in the range f_c - f_d to $f_c + f_d$, where f_d is the Doppler shift. The amount of spectral broadening depends on f_d which is a function of the relative velocity of the mobile, and the angle θ between the direction of motion of the mobile and direction of arrival of the scattered waves. If the baseband signal bandwidth is much greater than

B_d the effects of Doppler spread are negligible at the receiver. This is a slow fading channel [6-8].

2.10 Coherence time

Coherence time T_c is the time domain dual of Doppler spread and is used to characterize the time varying nature of the frequency depressiveness of the channel in the time domain. The Doppler spread and coherence time are inversely proportional to one another. The Doppler spread and coherence time are inversely proportional to one another. That is,

$$T_c \approx \frac{1}{f_m} \tag{2.1}$$

Coherence time is actually a statistical measure of the time duration over which the channel impulse response is essentially invariant, and quantifies the similarity of the channel response at different time. In other words, coherence time is the time duration over which two received signals have a strong potential for amplitude correlation. If the reciprocal bandwidth of the baseband signal is greater than the coherence time of the channel, then the channel will change during the transmission of the baseband message, thus causing distortion at the receiver.

If the coherence time is defined as the time over which the time correlation function is above 0.5,

Then the coherence time is approximately, that's

$$T_c = \frac{9}{16\pi f_m} \tag{2.2}$$

Where f_m is the maximum Doppler shift given by $fm = v / \lambda$. In practice, (1) suggests a time duration during which a Rayleigh fading signal may fluctuate widely and (2) is often too restrictive. A popular rule of thumb for modern digital communications is to define the coherence time as the geometric mean of Equations (1) and (2). That is,

$$T_c = \sqrt{\frac{9}{16\pi {f_m}^2}} = \frac{0.423}{f_m} \tag{2.3}$$

The definition of coherence time implies that two signals arriving with a time separation greater than T_C are affected differently by the channel [6-8].

2.11 Rayleigh distribution channel

In mobile radio channels, the Rayleigh distribution is commonly used to describe the statistical time varying nature of the received envelope of a flat fading signal, or the envelope of an individual multipath component. It is well known that the envelope of the sum of two Quadrature Gaussian noise signals obeys a Rayleigh distribution [6-8].

When there is a dominant stationary (non-fading) signal component present, such as a line-of sight propagation path, the small-scale fading envelope distribution is Rician. In such a situation, random multipath components arriving at different angles are superimposed on a stationary dominant signal. At the output of an envelope detector, this has the effect of adding a dc component to the random multipath.

Just as for the case of detection of a sine wave in thermal noise, the effect of a dominant signal arriving with many weaker multipath signals gives rise to the Rician distribution. As the dominant signal becomes weaker, the composite signal resembles a noise signal which has an envelope that is Rayleigh. Thus, the Rician distribution degenerates to a Rayleigh distribution when the dominant component fades away [6-8].

This is used to describe the statistical time varying nature of the envelope of an individual multipath component in the wireless communication. The Rayleigh distribution is given by

$$P(r) = \frac{r^2}{\sigma^2} \exp\left(\frac{-r^2}{\sigma^2}\right) ; 0 \leq r \leq \infty \qquad (2.4)$$

Where, σ= rms value of the received signal

$r^2/2$ = instantaneous power

σ^2 = local average power of the received signal before detection

2.12 Additive white Gaussian noise (AWGN)

A basic and generally accepted model for thermal noise in communication channels, is the set of assumptions that the noise is additive so that, the received signal equals the transmit signal plus some noise, where the noise is statistically independent of the signal. The noise is white, so that, the power spectral density is flat, so the autocorrelation of the noise in time domain is zero for any non-zero time offset and the noise samples have a Gaussian distribution.

As depicted in equation(3), in an AWGN channel, the transmitted signal $r(t)$ is additively corrupted by a white Gaussian noise source $n(t)$, and the received signal $s(t)$ is given by:

$$s(t) = r(t) + n(t) \tag{2.5}$$

The white noise $n(t)$ is a real-valued zero-mean Wide Sense Stationary (WSS) random process with Gaussian Probability Density Function (PDF). The term 'white' is use in the sense that the Power Spectral Density (PSD) function of the noise is constant over the whole frequency-domain, and its Auto-Correlation Function (ACF) is a direct pulse at zero delay as illustrated in figure 1.2.

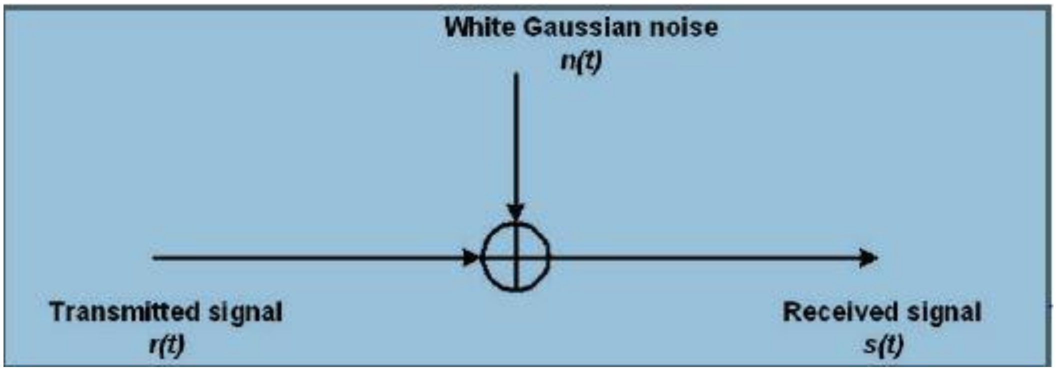

Fig 2.2:Pass band representation of AWGN channel

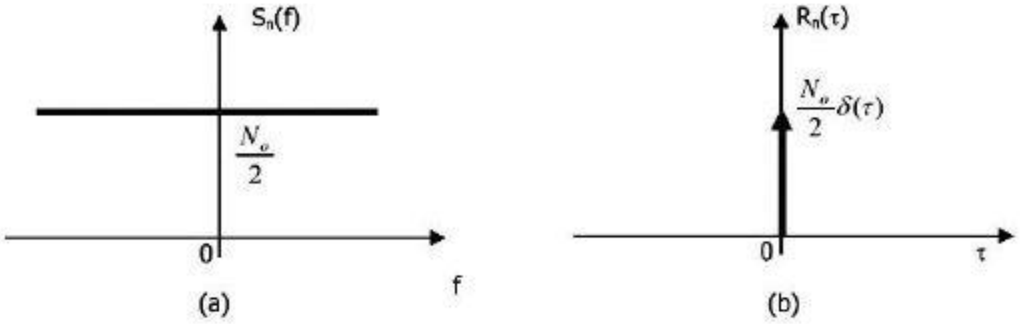

Fig 2.3:The wideband white noise: (a) The PSD, and (b) The ACF

Mostly it is also assumed that the channel is Linear and Time Invariant. The most basic results further assume that it is also frequency non-selective [7-9].

2.13 Bit error rate (BER)

BER is key parameters to use in accessing systems that is transmitted digital data one location to another location.BER is applicable in wireless radio data links, Ethernet and as well as fiber optic communications. But when radio data is transmitted through the wireless communication then there is a possibility errors can be introduced into the systems. If this so the integrity of the systems can be compromised. As a result, it is necessary to assess the performance of the system, and BER provides an ideal way in which this can be achieved. BER assesses the full end to end performance of a system including the transmitter, receiver and the medium between the two [9].

BER is defined as the rate at which errors occur in a transmission system.

In simple form,

$$\text{BER}=\frac{number\ of\ bits\ in\ error}{total\ number\ of\ bits\ sent}$$

BER expression is given by equation as,

$$BER=\int_0^{\infty} P_b\left(\frac{E}{r}\right)P(r)dr \tag{2.6}$$

Where, Pb(E/r) = the conditional error probability, *P(r)* = The pdf of the SNR.

Chapter 3

Methodology

In this paper we analysis wireless signal in Rayleigh fading channel by using BPSK and QPSK. Here we calculate SNR of the transmitted signal and compare with the SER at the receiver end. To determine bit error rate we use probability density function and Q-function. The steps are described as following: [10-11]

3.1 BER for BPSK Modulation

In a BPSK system the received signal can be written as:

$$y = x + n \tag{1}$$

Where $x \in \{-A, A\}$, $n \sim CN(O, \sigma^2)$ and $\sigma^2 = N_0$. The real part of the above equation is; $y_{re} = x + n_{re}$, Where $n_{re} \sim N(0, \sigma^2/2) = N(0, N_0/2)$. In BPSK constellation $d^2{}_{min} = 2A$ and γ_b is defined as E_b/N_0 and sometimes it is called SNR per bit. With this definition we have,

$$\gamma_b = \frac{E_b}{N_0} = \frac{A^2}{N_0} = \frac{d^2{}_{\min}}{4N_0} \tag{2}$$

So the bit error probability is:

$$P_b = P\{n > A\} = \int_A^\infty \frac{1}{\sqrt{2\pi\sigma^2/2}} e^{-\frac{x^2}{2\sigma^2/2}} \tag{3}$$

This equation can be simplified using Q-function as:

$$P_b = Q\left(\sqrt{\frac{d^2{}_{\min}}{2N_0}}\right) = Q\left(\frac{d_{\min}}{\sqrt{2N_0}}\right) = Q\left(\sqrt{2\gamma_b}\right) \tag{4}$$

Where, the Q function is defined as:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-\frac{x^2}{2}} dx \tag{5}$$

3.2 BER for QPSK Modulation

QPSK modulation consists of two BPSK modulation on-in phase and quadrature components of the signal. The corresponding constellation is presented on figure.

The BER of each branch is the same as BPSK.

$$P_b = Q\left(\sqrt{2\gamma_b}\right) \tag{6}$$

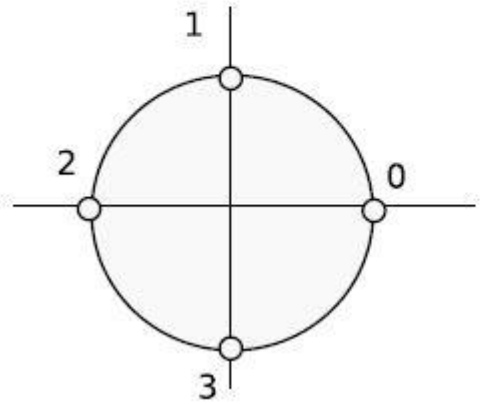

Figure 3.1: QPSK constellation

The approximations or exact values for SER have been following form

$$P_s(\gamma_s) \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right) \tag{7}$$

Where α_M and β_M depend on the type of approximation and the modulation type. In the table 1 the values for $\propto_M$ andβ_M both are summarized for common modulations.

We can also note that the bit error probability has the same from as for SER. It is:

$$p_b(\gamma_b) \approx \overline{\alpha}_M Q\left(\sqrt{\hat{\beta}_M \gamma_b}\right) \tag{8}$$

Where $\hat{\alpha}_M = \alpha_M / \log_2 M$ and $\hat{\beta}_M = \beta_M / \log_2 M$

Note: $\gamma_s = \frac{E_s}{N_0}$, $\gamma_b = \frac{E_b}{N_0}$, $\gamma_b = \frac{\gamma_s}{\log_2 M}$ and $P_b \approx \frac{p_s}{\log_2 M}$

3.3 SER AND BER over fading channel

PDF-based approach for binary signal

A fading channel can be considered as an AWGN with a variable gain. The gain itself is considered as $\Re\nu$ with a given pdf. So the average BER can be calculated by averaging BER for instantaneous SNR over the distribution of SNR;

$$P_b(E) = \int_0^\infty P_b(E/\gamma) p_\gamma(\gamma) d\gamma \tag{9}$$

The BER is expressed by a Q-function as seen in previous chapter.

$$P_b(E) = \int_0^\infty Q\left(\sqrt{2g\gamma}\right) p_\gamma(\gamma) d\gamma \tag{10}$$

Where, g=1 for the case of coherent BPSK.

Example 1: Rayleigh fading channel with coherent detection;

The received signal in a Rayleigh fading channel is the form;

$$y = hx + \omega \tag{11}$$

Where, h is the channel attenuation with the normal distribution $h \sim CN(0,1)$ and n is a white additive noise $\omega \sim CN(0, N_0)$. The coherent receiver constructs the following metric form the received signal.

$$h * y = |h|^2 x + h * \omega \tag{12}$$

Using BPSK modulation and since the information are real, only the real part of the equation is of interest. So the following sufficient statistic is used for decision at the receiver.

$$\Re\left\{\frac{h*}{|h|} y\right\} = |h| x + n$$

The noise n has the same statistics as $\Re\omega$ because $h*/|h| = \exp(j\theta)$ with θ uniformly distributed in(0,π),therefore n$\sim CN(0, \frac{N_0}{2})$. This equation shows that we have a normal AWGN channel with the signal scaled by |h|. The bit error probability as seen before for this case, given h, will be; $P_b = Q\left(\sqrt{2|h|^2 \gamma_b}\right)$

Now, we compute the SER by averaging this BER over the distribution of h. Since h is complex Gaussian, the distribution of $r = |h|^2$ will be exponential with;

$$P_r(r) = \frac{d}{dr}\left(P\left(h_r^{\,2} + h_i^{\,2} < r\right)\right)$$

$$= \frac{d}{dr}\left(\int_0^{2\pi}\int_0^{\sqrt{r}} \frac{1}{2\pi 1/2} e^{-x^2} x dx d\theta\right)$$

$$= \frac{d}{dr} e^{-r}$$

$$= e^{-r} \cup (r)$$

Therefore the signal-to-noise-ratio distribution $\gamma = |h|^2 \gamma_b$ will be;

$$p_\gamma = \frac{1}{\gamma_b} e^{-\gamma/\gamma_b}$$

The error probability can be calculated by:

$$P_b = \int_0^{\infty} Q\left(\sqrt{2\gamma}\right) p_{\gamma}(\gamma) d\gamma$$

$$= \int_0^{\infty} Q\left(\sqrt{2\gamma}\right) \frac{1}{\gamma_b} e^{-\gamma / \gamma_b} d\gamma$$

Using the following form of Q-function and MGF function, the integral can be calculated;

$$Q(x) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(-\frac{x^2}{2\sin^2\theta} \right) d\theta$$

$$p_b = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_b}{1+\gamma_b}} \right)$$

Example 2: Consider a SIMO system with L receive antennas. Each branch has a SNR per bit of γ_l and therefore the SNR at the output of MRC combiner is, $\gamma_l = \sum_{l=1}^{L} \gamma_l$ Suppose a Rayleigh channel, the pdf of SNR for each channel will be (supposing i.i.d. channels);

$$p\gamma_l(\gamma_l) = \frac{1}{\bar{\gamma}} e^{-\gamma_l / \ddot{\gamma}_{\bar{l}}}$$

At the output of combiner, the SNR follows the distribution of chi-square (or gamma) with L degrees of freedom;

$$p_{\gamma_l}(\gamma_l) = \frac{1}{(L-1)\bar{\gamma}^L} \gamma_t^{L-1} e^{-\gamma_L / \bar{\gamma}}$$

The average probability can be calculated using the integration by part and resulting in the following formula;

$$P_b(E) = \left(\frac{1-\mu}{2} \right)^L \sum_{l=0}^{L-1} \genfrac{}{}{0pt}{}{L-1+1}{l} \left(\frac{1+\mu}{2} \right)^l$$

Chapter 4

Result & Simulation

The code below plots a faded signal's power (versus sample number). The code also illustrates the syntax of the filter and Rayleigh channel functions and the state retention of the channel object. Notice from the output that number samples processed equals the number of elements in sig, the signal.

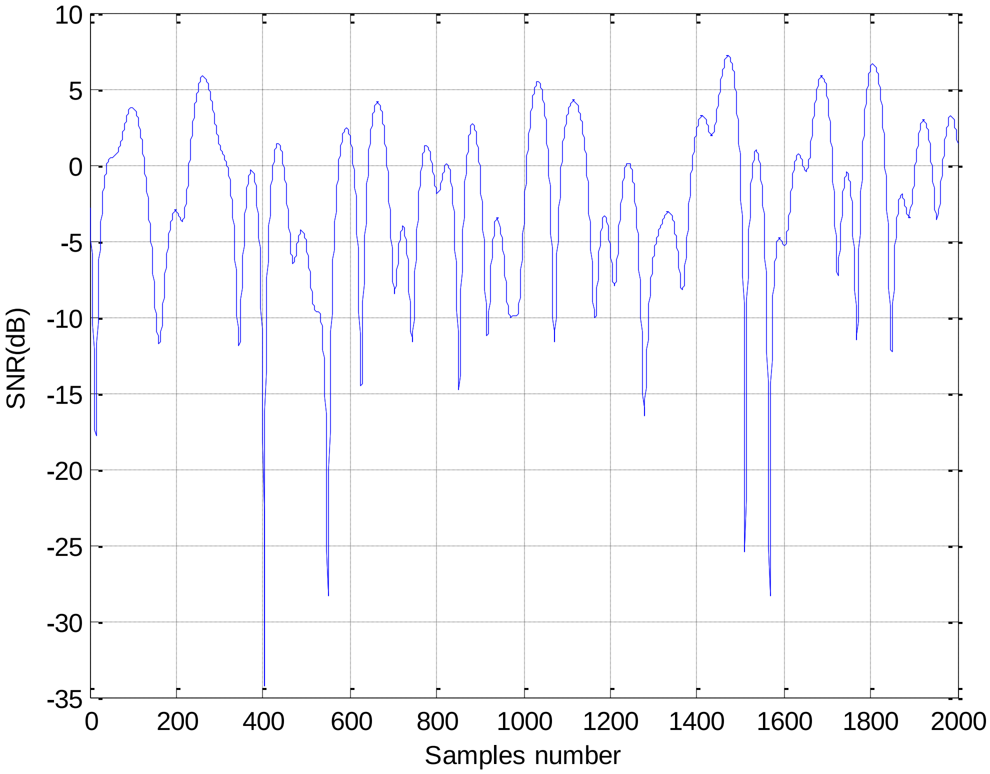

Figure 4.1: Power of faded signal, versus sample number.

In Rayleigh channel, we use 2000 sample at period 1.0000e to 004 with 100 of max Doppler shift shown in figure 4.1. The blue colored signal represents input sine wave. It is observed that when signal passes through the channel, signal gets phase shifted due environmental changes or obstacles along the path.

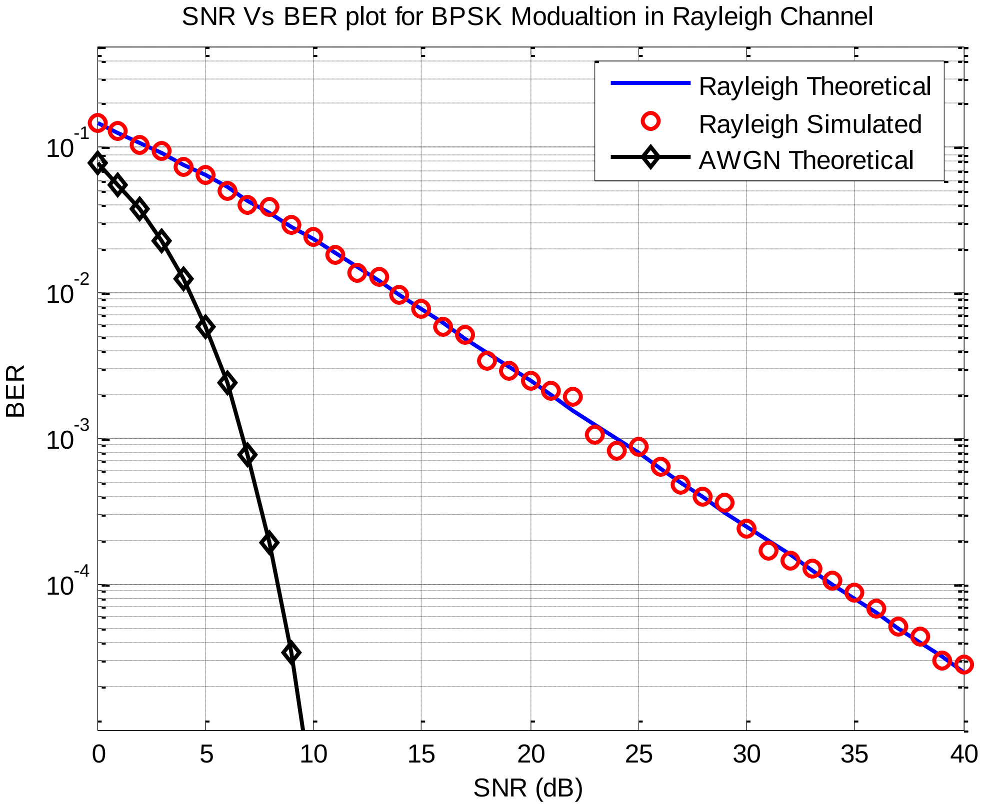

Figure 4.2: SNR vs. BER plot for BPSK modulation.

We observed in SNR vs. BER plot for BPSK shown in figure 4.2, the signal decreases in the AWGN channel at less than 10db and in Rayleigh channel it takes more time to decrease. We can increase the SNR by increasing the value of BER.

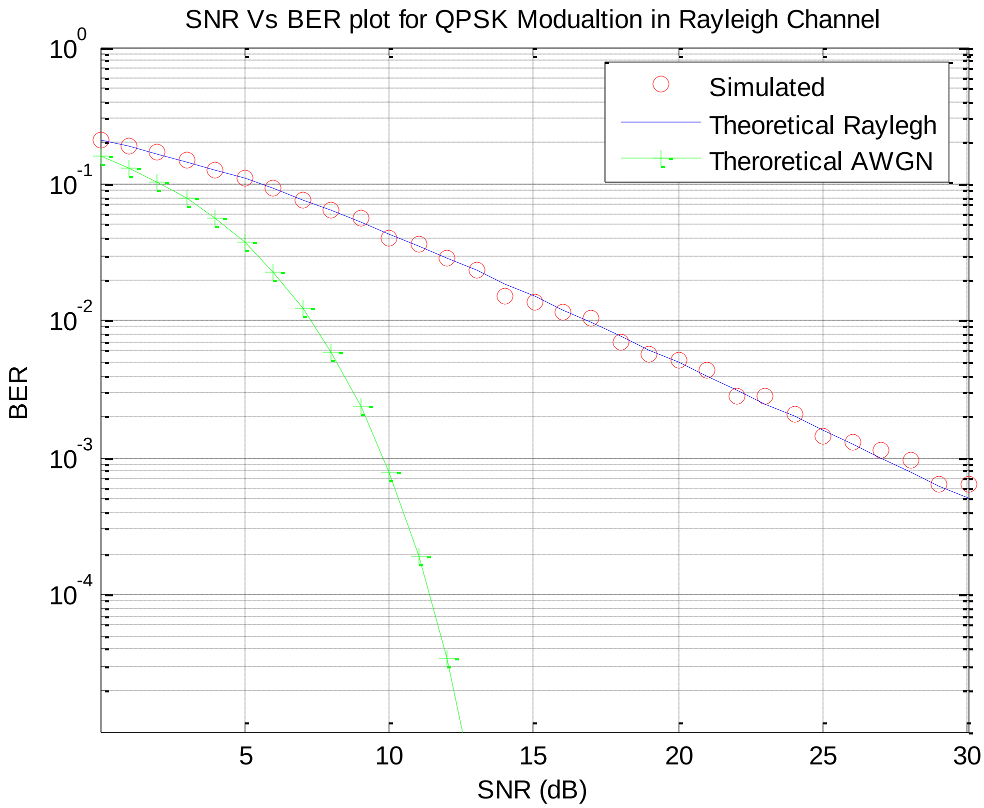

Figure 4.3: SNR vs. BER plot for QPSK modulation.

We also observed that SNR vs. BER plot for QPSK, where the signal decreases at between 10db to 15dbinAWGN channel and in Rayleigh channel it takes more time to decrease which is shown in figure 4.3.

In this paper, multipath fading channel model has been simulated. In BER vs. SNR plot, we have used BPSK and QPSK modulation to test the effect of different channels to the received signal. We have also compared and analyzed the improvement of Rayleigh channel with AWGN channel considering effect of BER and SNR on their performance in fading. The conclusion is that more accurate model is Rayleigh channel model because its BER curves have steepness and values more closely to theoretical analysis. In Rayleigh fading the amplitude gain is characterized by a distribution. For constant BER the value of SNR for Rayleigh is approximately same but in AWGN channel it decreases.

BPSK is able to transmit one bit per symbol, while QPSK transmits two bits per symbol. So QPSK can be used to double the data rate and still use the same bandwidth. For the same energy per symbol it is more likely that a QPSK symbol is wrongly decoded in comparison to a BPSK symbol. In BPSK modulation the signal space is represented by a single basis function:

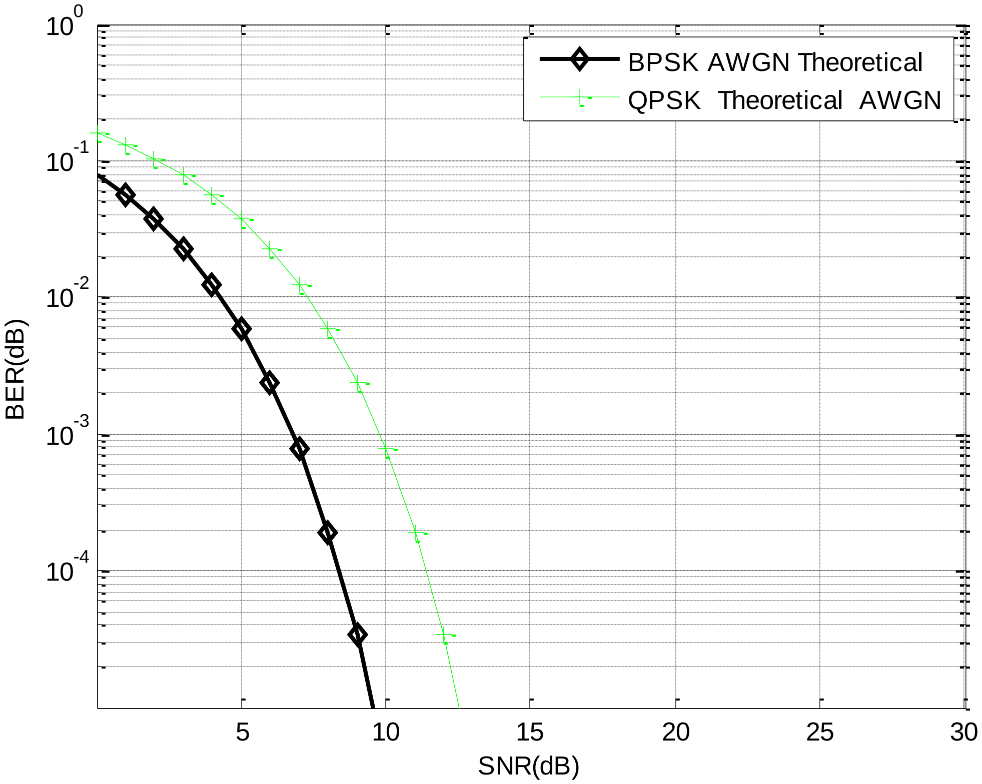

Figure 4.4: AWGN simulation in BPSK and QPSK.

In comparative study of bit error rate (BER) and signal to noise ratio (SNR) in BPSK and QPSK techniques under AWGN channel have been presented in figure 4.4. We get better performance of AWGN channel in QPSK than BPSK. For BPSK the signal decreases at 9dB in AWGN channel but it decreases at 10dB to 15dB in QPSK.

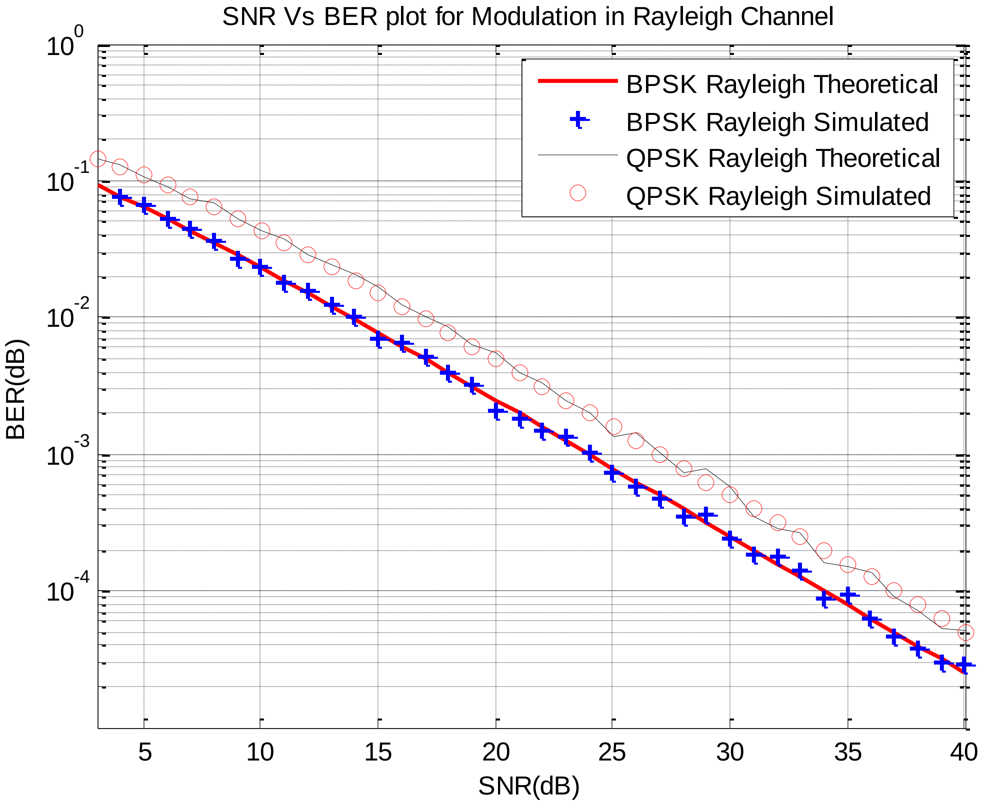

Figure 4.5: Comparison of BPSK and QPSK in Rayleigh fading channel.

In this figure comparative study of bit error rate (BER) & SNR (signal to noise ratio) in BPSK and QPSK techniques under Rayleigh fading channel has been presented in figure 4.5. For constant BER the value of SNR is better in QPSK than BPSK.

Chapter 5

Conclusion

In this paper, the comparative study of BPSK and QPSK show that the constant value of bit error rate (BER) is compared with the signal to noise ratio (SNR) under Rayleigh channel, the signal decreases less in QPSK than BPSK modulation. Because the transmission symbol rate of QPSK is two bit per symbol and in BPSK has one bit per symbol. The signal energy is spread out over the constellation. The symbols at the far ends will have less energy per symbol (Es) than the ones closer to the middle. We can improve more by using QAM modulation because of its high bit rate. QAM is a coherent modulation scheme that is commonly used in both wireless and wired communication to improve the signal performance. Using QAM instead of BPSK and QPSK in transmission will make the bit rate faster and efficient. It will also reduce the bit error rate. BER and SER are dependent on what channel the transmission is using. Both can be improved by using QAM.

Appendix

Appendix A

```
c = rayleighchan(1/10000,100);
sig = 1i*ones(2000,1);
y = filter(c,sig);
plot(20*log10(abs(y)))
```

The output and the plot follow.

```
c =


             ChannelType: 'Rayleigh'
       InputSamplePeriod: 1.0000e-004
         DopplerSpectrum: [1x1 doppler.jakes]
         MaxDopplerShift: 100
              PathDelays: 0
           AvgPathGaindB: 0
      NormalizePathGains: 1
            StoreHistory: 0
          StorePathGains: 0
               PathGains:  -0.8062 + 0.2648i
      ChannelFilterDelay: 0
    ResetBeforeFiltering: 1
     NumSamplesProcessed: 2000
```

Appendix B

```
clear all;


format long;


bit_count = 10000;


SNR = 0: 1: 40;


for aa = 1: 1: length(SNR)


    T_Errors = 0;
    T_bits = 0;


    while T_Errors < 100


        % Generate some random bits
        uncoded_bits  = round(rand(1,bit_count));


        % BPSK modulator
        tx = -2*(uncoded_bits-0.5);


        % Noise variance
        N0 = 1/10^(SNR(aa)/10);


        h = 1/sqrt(2)*[randn(1,length(tx)) + j*randn(1,length(tx))];
```

```
        % Send over Gaussian Link to the receiver
        rx = h.*tx + sqrt(N0/2)*(randn(1,length(tx))+i*randn(1,length(tx)));

        rx = rx./h;

        % BPSK demodulator at the Receiver
        rx2 = rx < 0;

        % Calculate Bit Errors
        diff = uncoded_bits - rx2;
        T_Errors = T_Errors + sum(abs(diff));
        T_bits = T_bits + length(uncoded_bits);

    end
    % Calculate Bit Error Rate
    BER(aa) = T_Errors / T_bits;
    disp(sprintf('bit error probability = %f',BER(aa)));

end

% Rayleigh Theoretical BER
SNRLin = 10.^(SNR/10);
theoryBer = 0.5.*(1-sqrt(SNRLin./(SNRLin+1)));

% Start Plotting
% Rayleigh Theoretical BER
figure(1);
semilogy(SNR,theoryBer,'-','LineWidth',2);
hold on;

% Simulated BER
```

```
figure(1);
semilogy(SNR,BER,'or','LineWidth',2);
hold on;
xlabel('SNR (dB)');
ylabel('BER');
title('SNR Vs BER plot for BPSK Modualtion in Rayleigh Channel');


% Theoretical BER
figure(1);
theoryBerAWGN = 0.5*erfc(sqrt(10.^(SNR/10)));
semilogy(SNR,theoryBerAWGN,'blad-','LineWidth',2);
legend('Rayleigh Theoretical','Rayleigh Simulated', 'AWGN Theoretical');
axis([0 40 10^-5 0.5]);
grid on;
```

Appendix C

```
clear all;
close all;
format long;
bit_count = 10000;
Eb_No = -3: 1: 30;
SNR = Eb_No + 10*log10(2);
For  aa = 1: 1: length(SNR)
        T_Errors = 0;
    T_bits = 0;
    while T_Errors < 100
    uncoded_bits  = round(rand(1,bit_count));
        B1 = uncoded_bits(1:2:end);
        B2 = uncoded_bits(2:2:end);
    qpsk_sig = ((B1==0).*(B2==0)*(exp(i*pi/4))+(B1==0).*(B2==1)...
            *(exp(3*i*pi/4))+(B1==1).*(B2==1)*(exp(5*i*pi/4))...
            +(B1==1).*(B2==0)*(exp(7*i*pi/4)));
       ray =sqrt(0.5*((randn(1,length(qpsk_sig))).^2+(randn(1,length(qpsk_sig))).^2));
        rx = qpsk_sig.*ray;
        % Noise variance
        N0 = 1/10^(SNR(aa)/10);
rx = rx + sqrt(N0/2)*(randn(1,length(qpsk_sig))+i*randn(1,length(qpsk_sig)));
        % Equaliser
        rx = rx./ray;
        B4 = (real(rx)<0);
        B3 = (imag(rx)<0);
        uncoded_bits_rx = zeros(1,2*length(rx));
        uncoded_bits_rx(1:2:end) = B3;
        uncoded_bits_rx(2:2:end) = B4;
         diff = uncoded_bits - uncoded_bits_rx;
```

```
        T_Errors = T_Errors + sum(abs(diff));
        T_bits = T_bits + length(uncoded_bits);
    end
    BER(aa) = T_Errors / T_bits;
end
  figure(1);
semilogy(SNR,BER,'or');
hold on;
xlabel('SNR (dB)');
ylabel('BER');
title('SNR Vs BER plot for QPSK Modualtion in Rayleigh Channel');
figure(1);
EbN0Lin = 10.^(Eb_No/10);
theoryBerRay = 0.5.*(1-sqrt(EbN0Lin./(EbN0Lin+1)));
semilogy(SNR,theoryBerRay);
grid on;
figure(1);
theoryBerAWGN = 0.5*erfc(sqrt(10.^(Eb_No/10)));
semilogy(SNR,theoryBerAWGN,'g-+');
grid on;
legend('Simulated', 'Theoretical Raylegh', 'Theroretical AWGN');
axis([SNR(1,1) SNR(end-3) 0.00001 1]);
```

Appendix D

```
clear all;
format long;
bit_count = 10000;
SNR = 0: 1: 40;
For aa = 1: 1: length(SNR)
T_Errors = 0;
T_bits = 0;
While T_Errors< 100
uncoded_bits  = round(rand(1,bit_count));

tx = -2*(uncoded_bits-0.5);
        N0 = 1/10^(SNR(aa)/10)
        % Rayleigh channel fading
        h = 1/sqrt(2)*[randn(1,length(tx)) + j*randn(1,length(tx))];
rx = h.*tx + sqrt(N0/2)*(randn(1,length(tx))+i*randn(1,length(tx)));
rx = rx./h;
        rx2 = rx< 0;
diff = uncoded_bits - rx2;
T_Errors = T_Errors + sum(abs(diff));
T_bits = T_bits + length(uncoded_bits);
end
BER(aa) = T_Errors / T_bits;
disp(sprintf('bit error probability = %f',BER(aa)));
end
figure(1);
theoryBerAWGN = 0.5*erfc(sqrt(10.^(SNR/10)));
semilogy(SNR,theoryBerAWGN,'blad-','LineWidth',2);
legend('BPSK AWGN Theoretical');
axis([0 40 10^-5 0.5]);
```

```
grid on;
hold on;

% QPSK simulation with Gray coding and simple Rayleigh (no LOS) multipath
% and AWGN included.

format long;
bit_count = 10000;
Eb_No = -3: 1: 30;
SNR= Eb_No + 10*log10(2);
For aa = 1: 1: length(SNR)
T_Errors = 0;
T_bits = 0;
    % Keep going until you get 100 errors
 While T_Errors< 100
uncoded_bits  = round(rand(1,bit_count));
        B1 = uncoded_bits(1:2:end);
        B2 = uncoded_bits(2:2:end);
qpsk_sig = ((B1==0).*(B2==0)*(exp(i*pi/4))+(B1==0).*(B2==1)...
            *(exp(3*i*pi/4))+(B1==1).*(B2==1)*(exp(5*i*pi/4))...
+(B1==1).*(B2==0)*(exp(7*i*pi/4)));
        % Variance = 0.5 - Tracks theoretical PDF closely
        ray = sqrt(0.5*((randn(1,length(qpsk_sig))).^2+(randn(1,length(qpsk_sig))).^2));
        % Include The Multipath
rx = qpsk_sig.*ray;
        % Noise variance
        N0 = 1/10^(SNR(aa)/10);
rx = rx + sqrt(N0/2)*(randn(1,length(qpsk_sig))+i*randn(1,length(qpsk_sig)));
rx = rx./ray;
        B4 = (real(rx)<0);
        B3 = (imag(rx)<0);
```

```
uncoded_bits_rx = zeros(1,2*length(rx));
uncoded_bits_rx(1:2:end) = B3;
uncoded_bits_rx(2:2:end) = B4;
        % Calculate Bit Errors
diff = uncoded_bits - uncoded_bits_rx;
T_Errors = T_Errors + sum(abs(diff));
T_bits = T_bits + length(uncoded_bits);
end
    % Calculate Bit Error Rate
BER(aa) = T_Errors / T_bits;
end
% Theoretical BER
figure(1);
theoryBerAWGN = 0.5*erfc(sqrt(10.^(Eb_No/10)));
semilogy(SNR,theoryBerAWGN,'g-+');
grid on;
xlabel('SNR(dB)');
ylabel('BER(dB)');
legend('BPSK AWGN Theoretical','QPSK  Theoretical  AWGN');
axis([SNR(1,1) SNR(end-3) 0.00001 1]);
hold on;
```

Appendix E

```
clear all;
format long;
bit_count = 10000;
SNR = 0: 1: 40;
For aa = 1: 1: length(SNR);
    % Initiate variables
T_Errors = 0;
T_bits = 0;
    % Keep going until you get 100 errors
While T_Errors< 100
uncoded_bits  = round(rand(1,bit_count));
        % BPSK modulator
tx = -2*(uncoded_bits-0.5);
        % Noise variance
        N0 = 1/10^(SNR(aa)/10);
        % Rayleigh channel fading
        h = 1/sqrt(2)*[randn(1,length(tx)) + j*randn(1,length(tx))];
        % Send over Gaussian Link to the receiver
rx = h.*tx + sqrt(N0/2)*(randn(1,length(tx))+i*randn(1,length(tx)));
rx = rx./h;
        rx2 = rx< 0 diff = uncoded_bits - rx2;
T_Errors = T_Errors + sum(abs(diff));
T_bits = T_bits + length(uncoded_bits);
end
    % Calculate Bit Error Rate
```

```
BER(aa) = T_Errors / T_bits;
disp(sprintf('bit error probability = %f',BER(aa)));
end
SNRLin = 10.^(SNR/10);
theoryBer = 0.5.*(1-sqrt(SNRLin./(SNRLin+1)));
figure(1);
semilogy(SNR,theoryBer,'r-','LineWidth',2);
hold on;
figure(1);
semilogy(SNR,BER,'+','LineWidth',2);
hold on;
xlabel('SNR (dB)');
ylabel('BER');
title('SNR Vs BER plot for BPSK Modualtion in Rayleigh Channel');
figure(1);
axis([0 40 10^-5 0.5]);
grid on;

% QPSK simulation with Gray coding and simple Rayleigh (no LOS) multipath
% and AWGN included
format long;
bit_count = 10000;
Eb_No = 0: 1: 40;
SNR = Eb_No + 10*log10(2);
For aa = 1: 1: length(SNR)
T_Errors = 0;
T_bits = 0;
```

```
While T_Errors< 100
uncoded_bits  = round(rand(1,bit_count));
      B1 = uncoded_bits(1:2:end);
      B2 = uncoded_bits(2:2:end);
qpsk_sig = ((B1==0).*(B2==0)*(exp(i*pi/4))+(B1==0).*(B2==1)...
         *(exp(3*i*pi/4))+(B1==1).*(B2==1)*(exp(5*i*pi/4))...
+(B1==1).*(B2==0)*(exp(7*i*pi/4)));
      % Variance = 0.5 - Tracks theoritical PDF closely
      ray = sqrt(0.5*((randn(1,length(qpsk_sig))).^2+(randn(1,length(qpsk_sig))).^2));
rx = qpsk_sig.*ray;
      N0 = 1/10^(SNR(aa)/10);
rx = rx + sqrt(N0/2)*(randn(1,length(qpsk_sig))+i*randn(1,length(qpsk_sig)));
rx = rx./ray;
      % QPSK demodulator at the Receiver
      B4 = (real(rx)<0);
      B3 = (imag(rx)<0);
uncoded_bits_rx = zeros(1,2*length(rx));
uncoded_bits_rx(1:2:end) = B3;
uncoded_bits_rx(2:2:end) = B4;
diff = uncoded_bits - uncoded_bits_rx;
T_Errors = T_Errors + sum(abs(diff));
T_bits = T_bits + length(uncoded_bits);
end
BER(aa) = T_Errors / T_bits;
end
% Finally plot the BER Vs. SNR(dB) Curve on logarithmic scale
figure(1);
```

```
semilogy(SNR,BER,'k-');
hold on;
xlabel('SNR (dB)');
ylabel('BER');
title('SNR Vs BER plot for Modulation in Rayleigh Channel');
figure(1);
EbN0Lin = 10.^(Eb_No/10);
theoryBerRay = 0.5.*(1-sqrt(EbN0Lin./(EbN0Lin+1)));
semilogy(SNR,theoryBerRay,'or');
grid on;
figure(1);
grid on;
xlabel('SNR(dB)');
ylabel('BER(dB)');
legend('BPSK Rayleigh Theoretical','BPSK Rayleigh Simulated','QPSK Rayleigh Theoretical','QPSK Rayleigh Simulated');
axis([SNR(1,1) SNR(end-3) 0.00001 1]);

hold on;
```

Appendix F

[math]\phi(t)= \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t).[/math]

While in QPSK modulation the signal space is represented by two basis function:

[math]\phi_1(t)= \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t).[/math]

[math]\phi_2(t)= \sqrt{\frac{2}{T_b}}\sin(2\pi f_c t).[/math]

Reference

[1] B. Sklar, Digital Communications: Fundamentals and Applications, Ch. 4, Englewood Cliffs, NJ: Prentice Hall, 1988.

[2] H. L. Van Trees,Detection Estimation, and Modulation Theory, Part I,Ch. 4, New York: Wiley, 1968.

[3] J. G. Proakis, DigitalCommunications, Chapter 7, New York: McGraw-Hill, 1983.

[4] E. Biglieri, J. Proakis and S. Shamai,"Fading channels: Information-theoretic and communications aspects," IEEE Trans. Inform. Theory, 50-th anniversary issue, Oct. 1998.

[5] V,Panwar,S.Kumar,"Bit Error Rate (BER) Analysis of Rayleigh Fading Channels in Mobile Communication," IJMER.Vol.2, No.3, pp-796-798,June. 2012.

[6] T. S. Rappaport, Wireless Communications, Chs. 3 and 4, Upper Saddle River, NJ: Prentice Hall, 1996.

[7] Leon, W.C. 2002. Digital and Analog Communication System, 6th Ed. Pearson Education, INC, Singapore.

[8] J. D. Parsons. The Mobile Radio Propagation Channel. John Wiley & Sons Ltd., 2000.

[9] Zhang, Q.T."Probability of error for equal-gain combiners over rayleigh channels: some closed- form solutions" IEEE Trans. Commun. 45(3), 270–273 (1997).

[10] Barry, J.R., Lee, E.A., Messerschmitt, D.G.: Digital Communication. Kluwer Academic Publishers, Massachusetts (2004).

[11] Wireless Communications by Andrea Goldsmith.