Radiation Pattern and Beamwidth Control of Linear and Rectangular Array Antenna System

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Abstract

Antenna array is one of the most prominent techniques of achieving electronic steerable beam i.e. rotation and variation of width of beam as required. This paper depicts a complete theoretical analysis of linear array antenna with transformation of the radiation pattern from a broadside to end-fire direction with change of relative phase, radiation intensity over a spherical surface, implementation of Dolph-Tschebyscheff array in reduction of side lobe of liner and rectangular array and baemwidth control of rectangular array.

Key Words

Antenna array, broadside, endfire, radiation pattern, side lobe control and array factor

1. Introduction

The simplest dipole antennas radiate and receive waves equally well in all directions and are called omnidirectional antennas. An omnidirectional antenna has equal gain in all directions but a directional antenna, on the other hand, has more gain in certain directions and less in other directions. Antennas can also be constructed to have certain fixed, preferential directions called directional antenna like Yagi-Uda of TV reciver. Enlarging the dimension of single elements often leads to more directive characteristics. Another way to enlarge the dimension of the antennas, without necessarily increasing the size of the individual elements, is to form an assembly of radiating elements in an electrical and geometrical configuration. This new antenna, formed by multi-elements, is referred to as an array. In most cases, the elements of an array are identical; although it is not necessary, it is often convenient, simpler, and more practical [1]-[3].

The main objective of any array antenna system is to control the width of the main beam and its direction. The purpose of our present paper is to analyze the impact of array parameters on the above mentioned phenomena for both linear and rectangular array antenna systems.

It should be noted here that the same antenna may be used as a transmitting antenna or as a receiving antenna. The gain of an antenna remains the same in both cases. The gain of a receiving antenna indicates the amount of power it delivers to the receiver compared to an omnidirectional antenna.

The total field of array is determined by the vector addition of the fields radiated by the individual elements. This assumes that the current in each element is the same as that of the isolated element. This is usually not the case and depends on the separation between the elements. To provide every directive patterns it is necessary that the fields from the elements of the array interfere constructively (add) in the desired directions and interfere destructively (cancel each other) in the remaining space [4],[5]. Ideally this can be accomplished, but practically it can only be approached. The direction where the maximum gain would appear is controlled by adjusting the phase between different antennas. In an array of identical elements, there are five controls that can be used to shape the overall pattern of the antenna. These are: geometrical configuration of the overall array (linear, circular, rectangular, spherical, etc.), the relative displacement between the elements, the excitation amplitude of the individual elements, the excitation phase of the individual elements and the relative pattern of the individual elements

Section 2 of the paper deals with linear array antenna with rotation of beam from broadside to endfire direction; Sec. 3 gives the same analysis for rectangular array along with control of side lobe. Section 4 depicts the results of previous sections and, finally, section 5 concludes the paper.

2. Linear Array

Let us assume a linear array of N elements as in Fig. 1. The elements have identical amplitudes but each succeeding element has a β progressive phase lead current excitation relative to the preceding one. The array factor of such antenna is given by [6],

$$AF = \sum_{n=1}^{N} e^{j(n-1)\psi} \tag{1}$$

where $\psi = kd \cos \theta + \beta$, and θ is the physical angle in azimuthal plane.

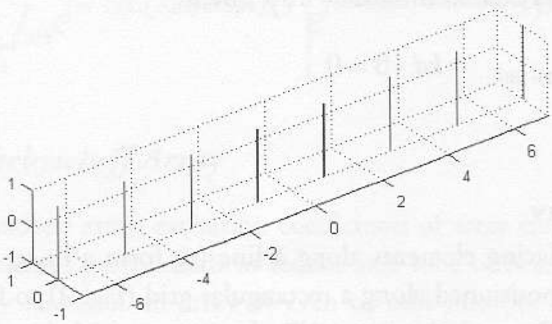

Figure 1: Linear array (N=8)

It is apparent that the amplitude and phase of AF can be controlled in uniform arrays by properly selecting the relative phase ψ between the elements; in non-uniform arrays, the amplitude as well as the phase can be used to control the formation and distribution of the total array factor.

2.1 *Broad side array*

In many applications it is desirable to have the maximum radiation of an array directed normal to the axis of the array. To optimize the design, the maximum of the single element and of the array factor should both be directed toward θ=90°. The requirements of the single elements can be accomplished by the judicious choice of the radiators and those of the array factor by the proper separation and excitation of the individual radiators employed in [6],[7]. The maxima of the array factor occurs when

$$\psi = kd \cos \theta + \beta = 0. \tag{2}$$

Since it is desired to have the maximum directed toward θ=90°, then

$$\psi = kd \cos \theta + \beta\big|_{\theta=90^0} \tag{3}$$

2.2 End-Fire Array

Instead of having the maximum radiation broadside to the axis of the array, it may be desirable to direct it along the axis of the array (end-fire).To direct the maximum toward $\theta=0^{\circ}$ or 180^{0},

$$\psi = kd\cos\theta + \beta\big|_{\theta=0^{0}} = kd + \beta = 0 \qquad (4)$$

$\Rightarrow \beta = kd.$

If the maximum is desired toward $\theta= 180^{\circ}$, then

$$\psi = kd\cos\theta + \beta\big|_{\theta=180^{0}} = kd + \beta = 0 \qquad (5)$$

$\Rightarrow \beta = kd.$

3. Planar Array

In addition to placing elements along a line (to form a linear array), individual radiators can be positioned along a rectangular grid (Fig. 2) to form a rectangular or planar array. Figure 2 shows an 8/8 planar array with bowtie antenna as the elements. Planar array provide additional variables which can be used to control and shape the pattern of the array. Planar arrays are more versatile and can provide more symmetrical patterns with lower side lobes [8]. In addition, they can be used to scan the main beam of the antenna toward any point in space. Applications include tracking radar, search radar, remote sensing, communications, and many others.

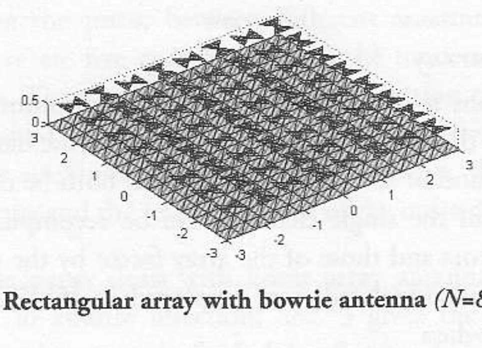

Figure 2: Rectangular array with bowtie antenna ***(N=8,M=8)***

3.1 *Rectangular Array factor*

If M elements are initially placed along the x-axis, its array factor can be written as:

$$AF = \sum_{m=1}^{M} I_{m1} e^{(m-1)(kd_x \sin\theta\cos\phi + \beta_x)}, \qquad (5)$$

where I_{mj} is the excitation coefficient of each element. The spacing and progressive phase shift between the elements along the x-axis are represented, respectively, by d_x and β_x. If N such array are placed next to each other in the y-direction, a distance d_y apart and with a progressive phase β_y, a rectangular array will be formed. The array factor for the entire planar array can be written as [9],

$$AF=\sum_{n=1}^{N} I_{1n}\left[\sum_{m=1}^{M} I_{m1} e^{(m-1)(kd_x \sin\theta\cos\phi+\beta_x)}\right] e^{j(n-1)(kd_y \sin\theta\cos\phi+\beta_y)}. \tag{6}$$

3.2 *Dolph-Tschebyscheff Array*

In Dolph-Tschebyscheff array, excitation coefficients of array elements are related to Dolph-Tschebyscheff polynomials to reduce side lobe of beam in a controlled manner. The array factor of an array of even or odd number of elements with symmetric amplitude excitation is nothing more than a summation of *M or M+1* cosine terms. Taking,

$$\begin{array}{ll}
z = \cos u, & \\
m=0 & \cos(mu) = 1 = T_0(z) \\
m=1 & \cos(mu) = z = T_1(z) \\
m=2 & \cos(mu) = 2z^2 - 1 = T_2(z) \\
m=3 & \cos(mu) = z^3 - 3z = T_3(z) \\
m=4 & \cos(mu) = 8z^4 - 8z^2 + 1 = T_4(z) \\
m=5 & \cos(mu) = 16z^5 - 20z^3 + 5z = T_5(z) \\
m=6 & \cos(mu) = 32z^6 - 48z^4 + 18z^2 - 1 = T_6(z) \\
m=7 & \cos(mu) = 64z^7 - 112z^5 + 56z^3 - 7z = T_7(z) \\
m=8 & \cos(mu) = 128z^8 - 256z^6 + 160z^4 - 32z^2 + 1 = T_8(z) \\
m=9 & \cos(mu) = 256z^9 - 576z^7 + 432z^5 - 120z^3 + 9z = T_9(z)
\end{array} \tag{2.10}$$

For the ratio of signal strength of main to side lobe R_0, z is replaced by z/z_0; where

$$z_0 = \frac{1}{2}\left[\left(R_0 + \sqrt{R_0^2 - 1}\right)^{\frac{1}{P}} + \left(R_0 - \sqrt{R_0^2 - 1}\right)^{\frac{1}{P}}\right] \tag{7}$$

and P is an integer equal to one less than the number of array elements. The array factor of linear array is as follows

$$(AF)_{2M} = \sum_{n=1}^{M} a_n \cos[(2n-1)u], \tag{8}$$

where $u = \pi d \cos\theta / \lambda$.

Equating (8) with corresponding Dolph-Tschebyscheff polynomials gives the coefficients a_n. Situation becomes very complicated for rectangular array as is shown in Appendix A.

3.3 *Beamwidth control*

Beamwidth of rectangular array antenna is defined as [10]-[12],

$$\Delta\theta = \tan^{-1}\left(\tan\theta_0 + \frac{1}{\alpha}\frac{d_y}{d_x}\right) - \tan^{-1}\left(\tan\theta_0 - \frac{1}{\alpha}\frac{d_y}{d_x}\right), \tag{9}$$

where θ_0 and $\Delta\theta$ are the direction of main lobe and beam width, and α is a constant known as beam width parameter. For $d_y=d_x$ above equation simplifies to,

$$\Delta\theta = \tan^{-1}\left(\tan\theta_0 + \frac{1}{\alpha}\right) - \tan^{-1}\left(\tan\theta_0 - \frac{1}{\alpha}\right). \tag{10}$$

For known value of θ_0 and $\Delta\theta$, the constant term α can be determined as,

$$\alpha = \frac{\tan\Delta\theta}{-1 + \sqrt{1 + \tan^2\Delta\theta + \tan^2\Delta\theta\tan^3\theta_0}}. \tag{11}$$

4. Result

Radiation intensity of two element end-fire array is shown in Fig. 3(a) on elevation plane, and in Fig. 3(b) on spherical surface; where color bar shows minimum (black) to maximum (white) intensity. Actually, the sphere in Fig. 3(b) gives the distribution of radial component of electric field. Array factor of the same antenna on elevation plane is shown in Fig. 3(a) for $d=\lambda/3$, f=75MHz. The same is done for broadside array in Fig. 4(a) and Fig. 4(b) respectively. Transformation of the radiation pattern from a broadside to end-fire as a function of phase shift for $N=2$, and N=4, taking $d=\lambda/4$, is shown in Fig. 5(a) and Fig. 5(b) respectively. Array factor of Dolph-Tschebysecheff linear array with major-to-minor lobe ratio *10dB, N=10, d=λ/2* (Polar coordinate system) is shown in Fig. 6. The same is done for rectangular array of 10x10 in Fig. 7. It is easily visualized that side lobe control is better for rectangular array compared to linear array. Variation of beamwith control parameter, α, against beamwith, taking direction of main lobe as a parameter, is shown in Fig. 8, which is a helping tool for getting antenna parameters for control of beamwidth of a smart antenna. It should be mentioned here that all the graphs of radiation pattern of this section are derived from Matlab Software.

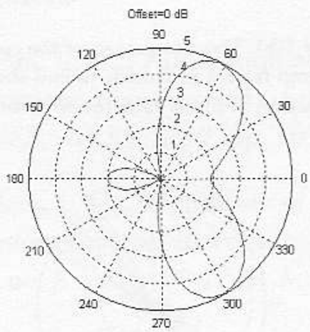

Figure 3(a): End-fire array with ***N=2,*** **Phase shift =-2π/3, d=λ/3,** ***f*****=75MHz**

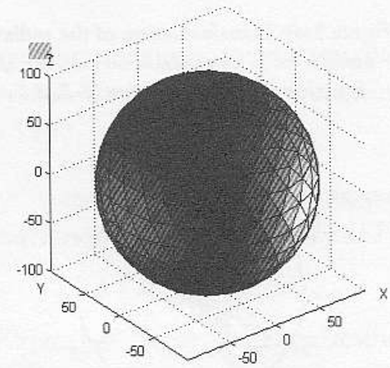

Figure 3(b): Radiation intensity over a spherical surface (end-fire)

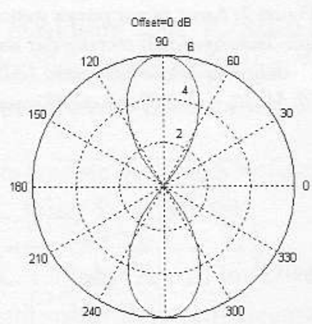

Figure 4(a): Radiation intensity over a spherical surface (broadside)

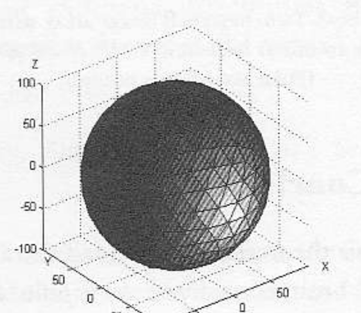

Figure 4(b): Broadside array with ***N=2,*** **Phase shift =*0, d=λ/3, f=75* MHz**

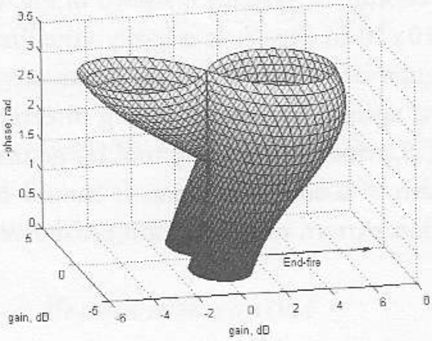

Figure 5(a): Transformation of the radiation pattern from a broadside to end-fire as a function of phase shift for *N=2, d=λ/4*

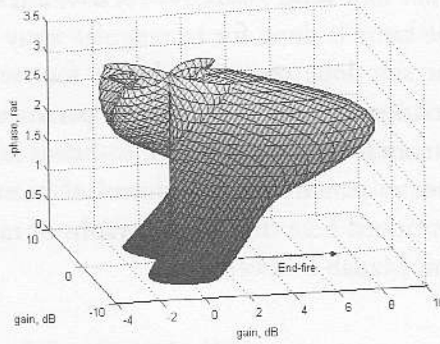

Figure 5(b): Transformation of the radiation pattern from a broadside to end-fire as a function of phase shift for *N=4, d=λ/4*

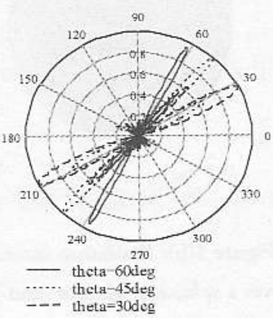

Figure 6: Array factor power pattern of Dolph-Tschebysecheff linear array with major-to-minor lobe ratio *10dB, N=10, d=λ/2* (Polar coordinate system)

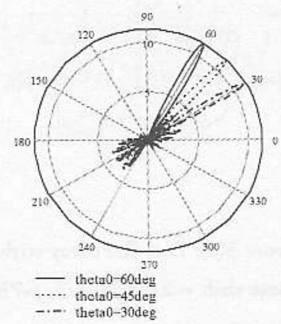

Figure 7: Array factor power pattern of Dolph-Tschebysecheff rectangular array with major-to-minor lobe ratio *10dB, N=10, M=10, d=λ/2* (Polar coordinate system)

5. Conclusion

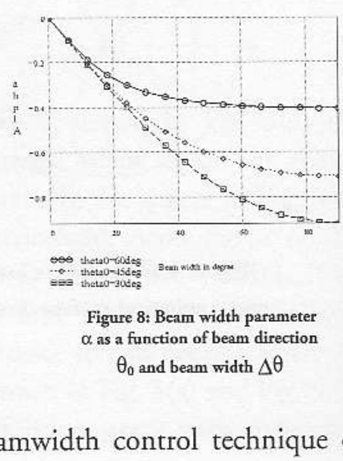

Figure 8: Beam width parameter α as a function of beam direction θ_0 and beam width $\Delta\theta$

In this theoretical analysis, all the array elements have been considered as dipole antenna. The smooth variation of radiation pattern of linear array from broadside to endfire, with variation of relative electrical phase, is a distinct finding of this paper. Side lobe and directivity control is visualized more prominently from polar radiation pattern; hence, rectangular array is recommended for smart antenna. More over, beamwidth control technique of rectangular array which is also depicted explicitly, would be a helping tool for a network planar in controlling traffic based on space division multiple access technique. This paper deals only with linear and rectangular array antenna but circular and cylindrical array could also be incorporated for comparison. It would be better if theoretical results of all the antennas discussed here could be compared with the practical result to validate the analysis.

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Appendix A

The array can be written as,

$$AF(\theta) = 4\sum_{m=1}^{N}\sum_{n=1}^{M} a_{m,n}\left[\cos\left[(2m-1)\frac{\pi d}{\lambda}(\sin(\theta_0)\cos(\phi) - \sin(\theta_0)\cos(\phi_0))\right]\right]$$
$$\times \cos\left[(2m-1)\frac{\pi d}{\lambda}(\sin(\theta)\sin(\phi) - \sin(\theta_0)\sin(\phi_0))\right],$$

where

$$a_{m,n} = \sum(-1)^{N-s}2\frac{(2N-1)}{N+s-1}C(N+s-1,2s-1)C(2s-1,s-n)\left[\frac{\cosh\left(\frac{1}{2N-1}a\cosh(R)\right)}{2}\right]^{2s-1}$$

and

$$P(x,y) = \begin{cases} x, & if \quad x \geq y \\ y, & if \quad x < y. \end{cases}$$

Here ϕ_0 and θ_0 are the directions of the main lobe in the azimuthal and the elevation plane.